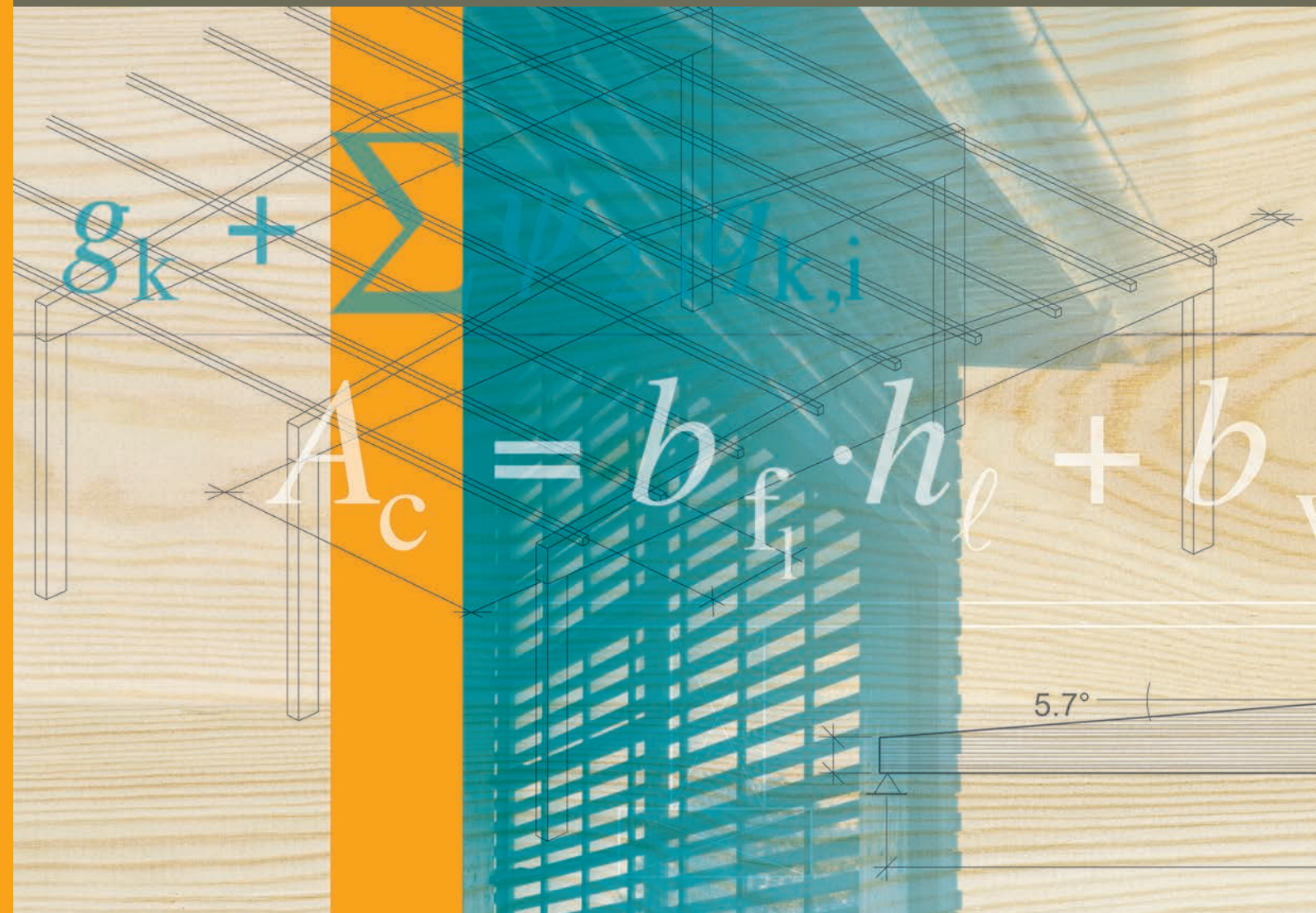


# Design of timber structures

Examples

## Volume 3

EDITION 2:2016



Design in the ultimate limit state (ULS)  
Design of timber joints  
Composite timber elements  
Horizontal stabilization  
Design for serviceability (SLS)

SWEDISH WOOD™



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Examples

Volume 3

EDITION 2:2016



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# Preface

This is the second revised edition of *Design of timber structures Volume 3, Examples* published in 2015. Rules and standards change in pace with the development of society, why a publication of this type has to be reviewed regularly.

The book series *Design of timber structures Volume 1–3* has been produced to make it easier for structural designers to calculate timber structures and it is adapted to Eurocodes and to Swedish application rules EKS 10 (BFS 2015:6). It is being used for higher education at universities and institutes.

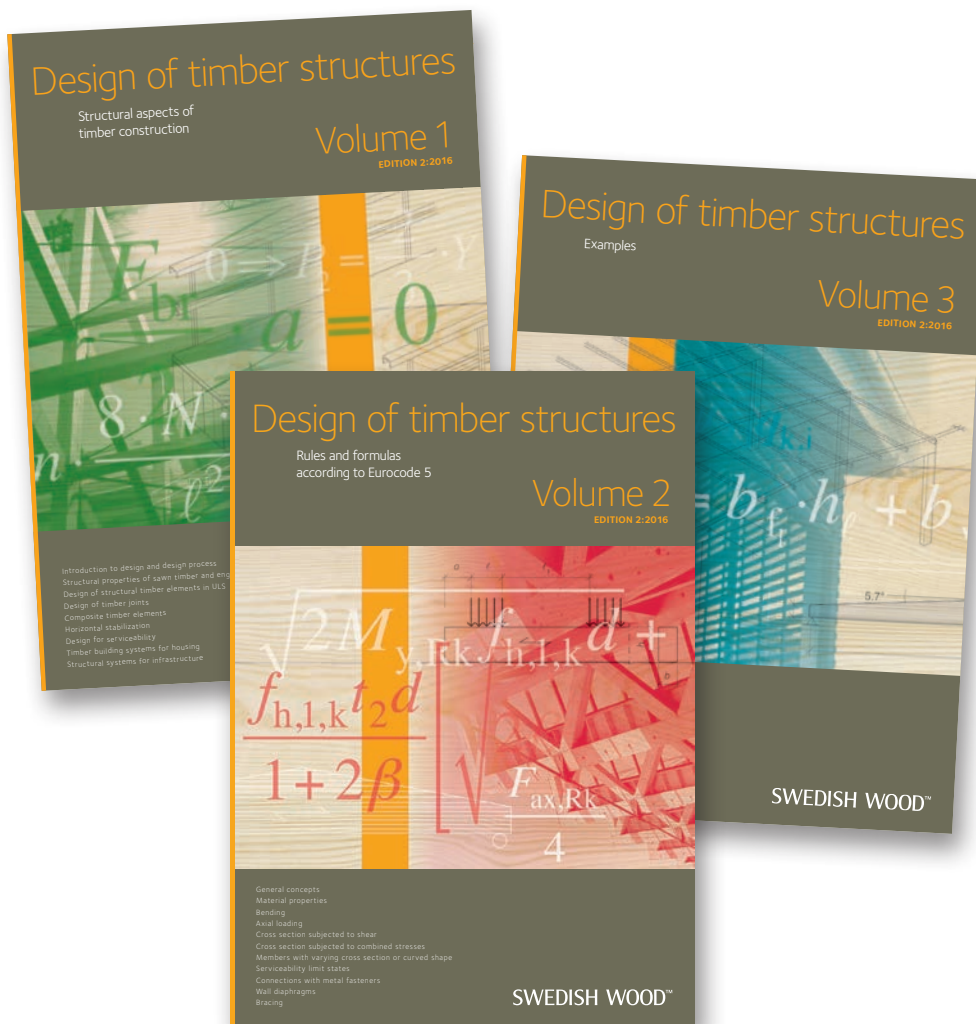
In the book series *Design of timber structures Volume 1–3* also *Volume 1, Structural aspects of timber construction* as well as *Volume 2, Rules and formulas according to Eurocode 5* are included. All three books are available in English and Swedish. Since the books are available in both languages and due to the nuanced content, our goal is that they will play a role for many users on different skill levels.

The solved examples in *Volume 3* has been compiled by the authors of each chapter in *Volume 1* respectively. The authors are Roberto Crocetti, Helena Lidelöw, Annika Mårtensson and Bert Norlin. Sven Thelandersson at the Faculty of Engineering at Lund University has been responsible for supervision and editing. The solutions to the examples are based on the rules in SS-EN 1995-1-1, but should not be regarded as an official interpretation of them. Professional application must be based on the original documents published by CEN with corresponding national application documents for Sweden.

More information about wood, glulam and timber construction can be found at [www.swedishwood.com](http://www.swedishwood.com).

Stockholm, October 2016

Eric Borgström  
**Swedish Wood**



## Design of timber structures

*Design of timber structures* Volumes 1–3 are adapted to Eurocode 5 and the Swedish application rules EKS 10 (BFS 2015:6).

- Volume 1: Structural aspects of timber construction
- Volume 2: Rules and formulas according to Eurocode 5
- Volume 3: Examples



### Dimensionering av träkonstruktioner

Del 1: Projektering av träkonstruktioner

Del 2: Regler och formler enligt Eurokod 5

Del 3: Dimensioneringsexempel

# Contents

The examples in this textbook are numbered after the matching chapter in *Design of timber structures Volume 1*. (Chapter 1, 2, 8 and 9 does not contain designing sections.)

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### 3 Design in the ultimate limit state (ULS)

#### Example 3.1 Design of a straight timber joist

A simply supported rectangular joist is subjected to characteristic permanent loading,  $g_k = 0,5 \text{ kN/m}^2$  and characteristic medium duration variable load  $q_k = 2,0 \text{ kN/m}^2$ . The clear span  $\ell$  is 4,5 m and the joists are spaced at 0,6 m centres.

The joist is part of an indoor floor. Service class 1 and safety class 2.

Design the joist in structural timber of strength class C24, for bending moment and shear load.

#### Solution

The characteristic value of bending strength is  $f_{m,k} = 24 \text{ MPa}$ .

Safety class 2:  $\gamma_d = 0,91$ .

The modification factor for load duration and service classes can be taken from Volume 2: Section 3.2, with load duration class M and service class 1 the factor  $k_{mod} = 0,8$ .

Material factor for structural timber,  $\gamma_M = 1,3$ .

Size factor for depth greater than 150 mm,  $k_h = 1,0$  (Volume 2: Section 3.3).

It can often be reasonable to let the factor  $k_h$  adopt the value of 1,0 even for smaller sizes of the joist.

Design value of the bending strength:

$$f_{m,d} = k_h \frac{k_{mod} f_{m,k}}{\gamma_M} = 1,0 \frac{0,8 \cdot 24}{1,3} = 14,8 \text{ MPa}$$

Design value of the load:

$$q_{dim} = \gamma_d \cdot \xi \cdot \gamma_g \cdot g_k + \gamma_d \cdot \gamma_q \cdot q_k = 0,91 \cdot 0,89 \cdot 1,35 \cdot 0,5 + 0,91 \cdot 1,5 \cdot 2,0 = 3,3 \text{ kN/m}^2$$

$$q_d = 0,6 \cdot 3,3 = 2,0 \text{ kN/m}$$

Design value of the bending moment:

$$M_{Ed} = q_d \ell^2 / 8 = 2,0 \cdot 4,5^2 / 8 = 5,1 \text{ kNm}$$

The required section modulus of the joist is given by:

$$W_{\text{erf}} = M_{\text{Ed}} / f_{\text{m,d}}$$

$$W_{\text{erf}} = \frac{M_{\text{Ed}}}{f_{\text{m,d}}} = \frac{5,1 \cdot 10^{-3}}{14,8} = 345 \cdot 10^{-6} \text{ m}^3$$

A common used width of a timber joist in Sweden is 45 mm. If this is chosen as the width  $b$  the required depth  $h = 214 \text{ mm}$  ( $\sqrt{(6 \cdot W_{\text{erf}})/b}$ ). A standard section is chosen  $45 \times 220 \text{ mm}$ . This requires that the joist is braced against lateral torsional buckling by for instance a floor graded particleboard.

### Check the shear capacity of the joist.

The design value of the shear force:

$$V_{\text{Ed}} = q_d \cdot \ell / 2 = 2,0 \cdot 4,5 / 2 = 4,5 \text{ kN}$$

Characteristic value of the shear strength:

$$f_{\text{v,k}} = 4,0 \text{ MPa}$$

Thereby is the design value of the shear strength given by:

$$f_{\text{v,d}} = \frac{k_{\text{mod}} \cdot f_{\text{v,k}}}{\gamma_{\text{M}}} = \frac{0,80 \cdot 4,0}{1,3} = 2,46 \text{ MPa}$$

The design value of the shear capacity is given by:

$$V_{\text{Rd}} = \frac{2}{3} A \cdot f_{\text{v,d}} \quad \text{kN}$$

According to the national annex in Sweden, at present EKS 10 (BFS 2015:6), the value of  $A$  should be determined on the basis of  $b_{\text{ef}}$  for a structural element subjected to bending moment. The effective width  $b_{\text{ef}}$  is given by:

$$b_{\text{ef}} = k_{\text{cr}} b$$

where  $k_{\text{cr}} = 3,0/f_{\text{v,k}} = 0,75$  for structural timber C24, not exposed to precipitation and solar radiation. This gives the following for the design value of shear capacity:

$$V_{\text{Rd}} = \frac{2}{3} \cdot k_{\text{cr}} A \cdot f_{\text{v,d}} = \frac{2}{3} \cdot 0,75 \cdot 0,045 \cdot 0,220 \cdot 2,46 \cdot 10^3 = 12,2 \text{ kN}$$

► **This shows that  $V_{\text{Ed}} < V_{\text{Rd}}$ , that is the joist has sufficient shear capacity.**

Remark: Some way to further reduce  $V_{\text{Ed}}$  is possible, see Volume 2: Chapter 6.



### Example 3.2 Bearing strength for the support a of timber joist

For the joist in Example 3.1 in this Volume, check the bearing strength at the end supports. The bearing length has been restricted to 66 mm at each end.

The check for bearing strength is given by:

$$\sigma_{c,90,d} \leq k_{c,90} \cdot f_{c,90,d}$$

The value of the bearing stress is given by:

$$A_{ef} = b \cdot (\ell_b + 0,03) = 0,045 \cdot (0,066 + 0,03) = 0,00432 \text{ m}^2$$

$$\sigma_{c,90,d} = \frac{P}{A_{ef}}$$

where  $b$  is the width of the beam,  $\ell_b$  is the actual bearing length and  $P = F_{c,90,d}$  is the force at the support.

$$b = 45 \text{ mm}$$

$$\ell_b = 66 \text{ mm}$$

$$P = \frac{q_d \ell}{2} = 4,5 \text{ kN} \quad (\text{according to Example 3.1})$$

Thus the bearing stress is:

$$\sigma_{c,90,d} = \frac{F_{c,90,d}}{A_{ef}} = \frac{4,5 \cdot 10^3}{0,00432 \cdot 10^6} = 1,04 \text{ MPa}$$

The characteristic value of the compressive strength is  $f_{c,90,k} = 2,5 \text{ MPa}$ .

Because  $g_k/q_k < 0,4$  it is possible to set  $k_{mod}$  and  $\gamma_M = 1,0$ , according to Volume 1: Section 3.1.3.

The design value of the compressive strength is:

$$f_{c,90,d} = \frac{k_{mod} f_{c,90,k}}{\gamma_M} = \frac{1,0 \cdot 2,5}{1,0} = 2,5 \text{ MPa}$$

The factor  $k_{c,90}$  is given by Volume 2: Section 5.2:

$$k_{c,90} = 1,5$$

The check is then given by:

$$\sigma_{c,90,d} = 1,04 \text{ MPa} \leq k_{c,90} \cdot f_{c,90,d} = 1,5 \cdot 2,5 = 3,8 \text{ MPa}$$

► **This shows that the bearing capacity at the end supports is sufficient.**

The example also shows that the fact that only a minor part of the beam is subjected to compression, allows quite high stresses compared to when a larger area is subjected to compression. If the joist is supported by a timber top plate, the bearing capacity of that of course also have to be checked.

### Example 3.3 Capacity of a notched timber beam

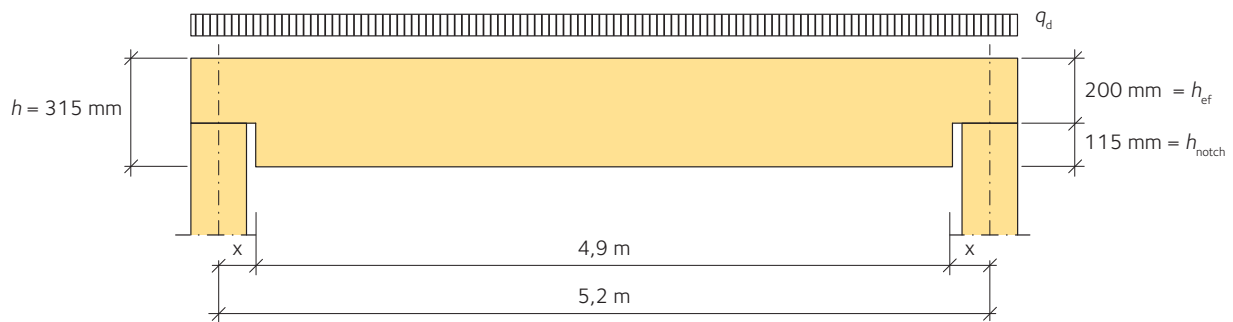


Figure 3.1

**Check the capacity of the beam in the figure above with regard to the ultimate limit state.** The beam is made of glulam GL30c and the cross section is  $90 \times 315 \text{ mm}$ . Service class 1, safety class 3 and load duration class M.

The design load has been determined to  $q_d = 5,1 \text{ kN/m}$ .

The design bending moment is:

$$M_{Ed} = \frac{q_d \ell^2}{8} = \frac{5,1 \cdot 5,2^2}{8} = 17,2 \text{ kNm}$$

The design shear force is:

$$V_{Ed} = \frac{q_d \ell}{2} = \frac{5,1 \cdot 5,2}{2} = 13,3 \text{ kN}$$

The characteristic value of the bending strength is  $f_{m,k} = 30 \text{ MPa}$ .

The modification factor for load duration and service classes can be taken from Volume 2: Section 3.2, with load duration class M and service class 1 the factor  $k_{mod} = 0,8$ .

Material factor for glulam,  $\gamma_M = 1,25$ .

Size factor for depths  $231 \text{ mm} \leq h < 600 \text{ mm}$ ,  $k_h = (600/315)^{0,1} = 1,07$  (Volume 2: Section 3.3).

Design value of the bending strength:

$$f_{m,d} = k_h \frac{k_{mod} f_{m,k}}{\gamma_M} = 1,07 \frac{0,8 \cdot 30}{1,25} = 20,5 \text{ MPa}$$

namely the bending moment is limited by:

$$M_{Rd} = f_{m,d} W = 20,5 \cdot 10^3 \cdot \frac{0,09 \cdot 0,315^2}{6} = 30,5 \text{ kNm}$$

namely the moment capacity is sufficient, provided that the beam is not subject to lateral buckling.

Characteristic value of the shear strength  $f_{v,k} = 3,5$  MPa.

Thereby is the design value of the shear strength given by:

$$f_{v,d} = \frac{k_{mod} \cdot f_{v,k}}{\gamma_M} = \frac{0,80 \cdot 3,5}{1,25} = 2,24 \text{ MPa}$$

The design value of the shear capacity is given by:

$$V_{Rd} = \frac{2}{3} A \cdot f_{v,d} \text{ kN}$$

According to the national annex in Sweden, at present EKS 10 (BFS 2015:6), the value of  $A$  should be determined on the basis of  $b_{ef}$  for a structural element subjected to bending moment. The effective width  $b_{ef}$  is given by:

$$b_{ef} = k_{cr} b$$

where  $k_{cr} = 3,0/f_{v,k} = 0,86$  for glulam, not exposed to precipitation and solar radiation. This gives the following for the design value of shear capacity:

$$V_{Rd} = \frac{2}{3} k_{cr} A \cdot f_{v,d} = \frac{2}{3} \cdot 0,86 \cdot 0,09 \cdot 0,315 \cdot 2,24 \cdot 10^3 = 36,3 \text{ kN}$$

► **This shows that  $V_{Ed} < V_{Rd}$  that is the beam has sufficient shear capacity.**

Now the notched areas have to be checked. In Eurocode 5: Section 6.5.2, it is stated that the following should be verified (also see Volume 2: Section 8.3):

$$\tau_d = \frac{1,5V_{Ed}}{b_{ef}h_{ef}} \leq k_v f_{v,d}$$

where  $h_{ef}$  is the reduced depth of the beam in the notch and  $k_v$  is a reduction factor defined as follows for beams notched on the same side as the support.

$$k_v = \min \left\{ \begin{array}{l} 1 \\ \frac{k_n \left( 1 + \frac{1,1i^{1,5}}{\sqrt{h}} \right)}{\sqrt{h} \left( \sqrt{\alpha(1-\alpha)} + 0,8 \frac{x}{h} \sqrt{\frac{1}{\alpha} - \alpha^2} \right)} \end{array} \right.$$

where:

- $i$  is the notch inclination  
 $h$  is the beam depth in mm  
 $x$  is the distance from the line of action of the support reaction to the corner of the notch, in mm

$$\alpha = \frac{h_{\text{ef}}}{h}$$

$$k_n = 6,5 \text{ for glulam.}$$

Width of the beam	$b = 90 \text{ mm}$
Depth of the beam	$h = 315 \text{ mm}$
Notch depth at each end of beam	$h_{\text{notch}} = 115 \text{ mm}$
Notch inclination	$i = 0$
Effective beam depth at each end	$h_{\text{ef}} = 200 \text{ mm}$
Ratio of $h_{\text{ef}}/h$	$\alpha = h_{\text{ef}}/h = 0,63$
Length of the notch from the centre line of the end support	$x = 150 \text{ mm}$

$$k_v = \min \left\{ \begin{array}{l} 1 \\ 6,5 \cdot \left( 1 + \frac{1,1 \cdot 0^{1,5}}{\sqrt{315}} \right) \\ \frac{6,5 \cdot \left( 1 + \frac{1,1 \cdot 0^{1,5}}{\sqrt{315}} \right)}{\sqrt{315} \left( \sqrt{0,63(1-0,63)} + 0,8 \cdot \frac{150}{315} \sqrt{\frac{1}{0,63} - 0,63^2} \right)} \end{array} \right. = 0,41$$

The shear stress is:

$$\tau_d = \frac{1,5 \cdot 13,3 \cdot 10^{-3}}{0,86 \cdot 0,09 \cdot 0,2} = 1,29 \text{ MPa}$$

while the reduced shear strength is given by:

$$k_v f_{v,d} = 0,41 \cdot 2,24 = 0,92 \text{ MPa}$$

► **This means that the notched beam not has a sufficient capacity.**

Further calculations give that it could be possible to have a notch corresponding to 16 percent of the total depth, in order to still have the sufficient shear capacity. Alternatively the notches could be reinforced for example with wood screws.

### Example 3.4 Lateral torsional buckling check of a glulam beam

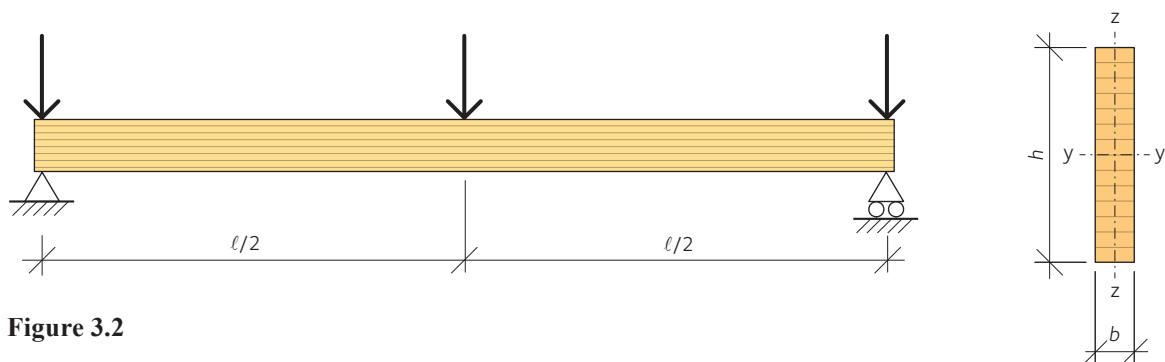


Figure 3.2

A simply supported beam is loaded with point loads (coming from columns standing on the beam). The point loads are of the size  $G_k = 5,25$  kN characteristic action (permanent duration) and  $Q_k = 14,45$  kN variable action (medium-term duration). The beam length is  $\ell = 10$  m. The beam has the dimensions  $h = 630$  mm and  $b = 115$  mm and is made of glulam GL30c.

**Check if the beam dimension is enough in service class 1 and safety class 3.**  
(Include the effect of lateral torsional buckling)

#### 3.4.1 Geometric properties of the beam

Width of the beam	$b = 115$ mm	
Depth of the beam	$h = 630$ mm	
Clear span of the beam	$\ell = 10\,000$ mm	
Section modulus of the beam about the y-y axis	$W_y = \frac{b \cdot h^2}{6}$	$W_y = 7,61 \cdot 10^6$ mm <sup>3</sup>

#### 3.4.2 Glulam properties

##### Glulam GL30c:

Characteristic bending strength	$f_{m,k} = 30$ N/mm <sup>2</sup>
Characteristic shear strength	$f_{v,k} = 3,5$ N/mm <sup>2</sup>
Characteristic compression strength perpendicular to the grain	$f_{c,90,k} = 2,5$ N/mm <sup>2</sup>
5 <sup>th</sup> percentile modulus of elasticity parallel to the grain	$E_{0,05} = 10,8$ kN/mm <sup>2</sup>
5 <sup>th</sup> percentile modulus of shear parallel to the grain	$G_{05} = 0,54$ kN/mm <sup>2</sup>

Mean modulus of elasticity parallel to the grain	$E_{0,\text{mean}} = 13,0 \text{ kN/mm}^2$
Mean shear modulus	$G_{\text{mean}} = 0,65 \text{ kN/mm}^2$
Mean density of the beam	$\rho_m = 430 \text{ kg/m}^3$

### 3.4.3 Partial safety factors

EKS 10: Table B-3 for ULS:

Permanent actions	$\gamma_G = 1,35$
Variable actions	$\gamma_Q = 1,5$
Volume 2: Table 3.1	
Material factor for glulam	$\gamma_M = 1,25$
Safety class 3	$\gamma_d = 1,0$

### 3.4.4 Actions

Self weight of the beam  
(often  $5 \text{ kN/m}^3$  is assumed as self weight for timber in general when designing)

$$g_{k,\text{selfwt}} = b \cdot h \cdot g \cdot \rho_m$$

$$g_{k,\text{selfwt}} = 0,31 \text{ kN/m}$$

Design action from the self weight of the beam

$$g_{d,\text{selfwt}} = \gamma_d \cdot \zeta \cdot \gamma_G \cdot g_{k,\text{selfwt}}$$

$$g_{d,\text{selfwt}} = 0,37 \text{ kN/m}$$

Characteristic permanent action from the point load

$$G_{k,p} = 5,25 \text{ kN}$$

Characteristic variable action from the point load

$$Q_{k,p} = 14,45 \text{ kN}$$

Design action from the point load for the critical load case at the ULS (Eurocode 0: Equation 6:10 and EKS 10: Table B-3) using the unfavourable condition variable action

$$F_{d,p} = \gamma_d \cdot \zeta \cdot \gamma_G \cdot G_{k,p} + \gamma_d \cdot \gamma_Q \cdot Q_{k,p}$$

$$F_{d,p} = 1,0 \cdot 0,89 \cdot 1,35 \cdot 5,25 + 1,0 \cdot 1,5 \cdot 14,45$$

$$= 27,98 \text{ kN}$$

### 3.4.5 Modification factors

Factor for medium duration  
loading M and service class 1  
(Volume 2: Table 3.2)

$$k_{\text{mod}} = 0,8$$

Size Factor for depth greater  
than 600 mm (Volume 2: Section 3.3)

$$k_{\text{h}} = 1,0$$

Lateral stability of the beam  
(Volume 2: Chapter 4)

$$k_{\text{crit}}$$

Effective length of the beam  
– adopt case for the most  
critical condition – the point load  
(concentrated load) at mid-span  
(Volume 2: Table 4.1)

$$l_{\text{ef}} = 0,8 l$$

$$l_{\text{ef}} = 8 \text{ m}$$

Critical bending stress  
(Volume 2: Chapter 4 or  
Eurocode 5: Equation 6.32)

$$\sigma_{\text{m,crit}} = \frac{0,78 \cdot b^2}{h l_{\text{ef}}} E_{0,05}$$

$$\sigma_{\text{m,crit}} = \frac{0,78 \cdot 115^2}{630 \cdot (0,8 \cdot 10000 + 2 \cdot 630)} \cdot 10800 = 19,1 \text{ N/mm}^2$$

Relative slenderness for bending  
(Volume 2: Chapter 4 or  
Eurocode 5: Equation 6.30)

$$\lambda_{\text{rel,m}} = \sqrt{\frac{f_{\text{m,k}}}{\sigma_{\text{m,crit}}}}$$

$$\lambda_{\text{rel,m}} = 1,25$$

Lateral stability factor  
(Volume 2: Chapter 4 or  
Eurocode 5: Equation 6.34)

$$k_{\text{crit}} = \begin{cases} 1 & \text{if } \lambda_{\text{rel,m}} \leq 0,75 \\ 1,56 - 0,75 \lambda_{\text{rel,m}} & \text{if } 0,75 < \lambda_{\text{rel,m}} \leq 1,4 \\ \frac{1}{\lambda_{\text{rel,m}}^2} & \text{if } \lambda_{\text{rel,m}} > 1,4 \end{cases}$$

$$k_{\text{crit}} = 0,62$$



### 3.4.6 Bending capacity

The design load case at the ULS will be the result of a combination of self weight of the beam plus the combined permanent and variable point loads at mid span:

Design bending moment  $M_{Ed} = 74,6 \text{ kNm}$

Design bending strength  $f_{m,y,d} = 19,20 \text{ N/mm}^2$

Design bending strength taking lateral torsional buckling effect into account (Volume 2: Chapter 4)  $M_{Rd} = f_{m,y,d} \cdot W_y \cdot k_{crit} \quad M_{Rd} = 90,6 \text{ N/mm}^2$

► **Bending capacity is satisfactory for glulam, 115 × 630 GL30c.**

### 3.4.7 Shear capacity

The design load case will be the result of combination of self weight of the beam plus the combined permanent and variable point loads at mid span:

Design value for end shear force  $V_d = 15,8 \text{ kN}$

Modification factor for shear (crack factor)  $k_{cr} = 3,0/3,5 = 0,86$

Effective width for shear  $b_{ef} = k_{cr} \cdot b \quad b_{ef} = 99 \text{ mm}$

Design shear stress (Volume 2: Chapter 6)  $\tau_d = 0,38 \text{ N/mm}^2$

Design shear strength (Volume 2: Section 3.1)  $f_{v,d} = 2,24 \text{ N/mm}^2$

► **The beam dimension is sufficient.**

### Example 3.5 Design of a double tapered beam (ULS and SLS)

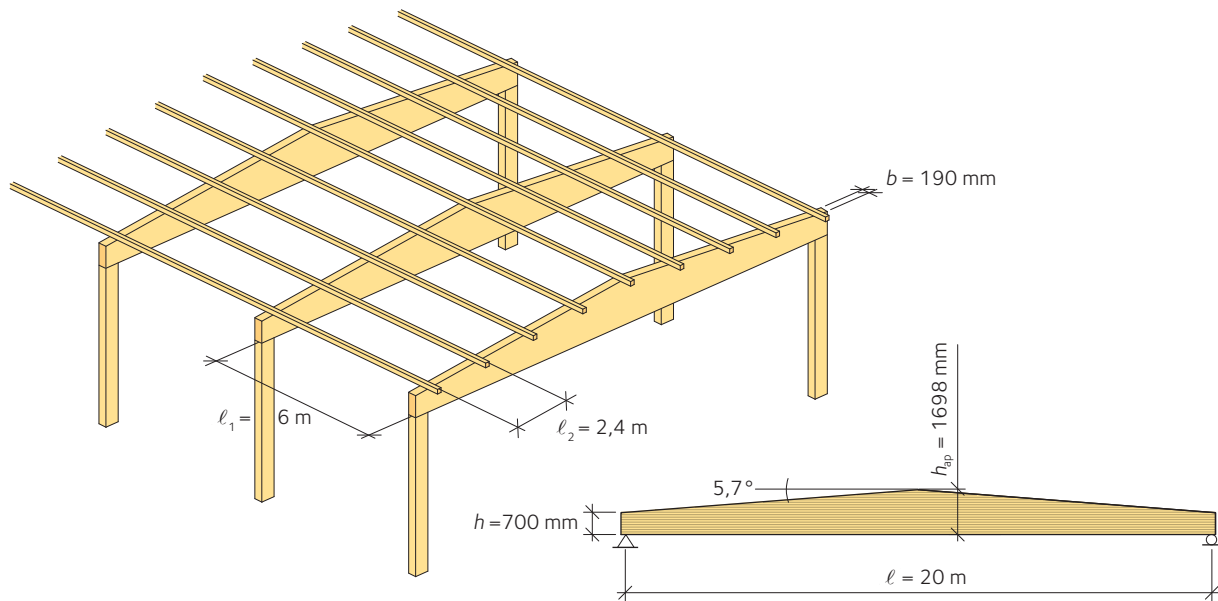


Figure 3.3: Geometry of the beam.

#### Loads acting on the beam

The loads considered for the design of the double tapered beam are the following: beam self weight, roof dead load and snow load. The wind load can be neglected. The centre-to-centre distance between the primary beams is  $\ell_1 = 6$  m. However, the influence area for a beam is considered 10 percent larger than  $\ell \times \ell_1$ , to take into account the effect of the continuity of the purlins over the primary beams. The snow load is reduced to take into account the roof shape factor  $\mu = 0,8 + 5,7/20 \cdot 0,3 = 0,8855$ . Here as an approximation on the safe side we calculate with the snow load on the leeward side according to EKS 10 as uniformly distributed along the whole beam length.

Table 3.1

Load type	Uniformly distributed load [kN/m <sup>2</sup> ]	Uniformly distributed load [kN/m]
Beam self weight		$g_{1k} = 1,10$
Roof dead load	0,6	$g_{2k} = 3,96$
Snow	1,5	$s = 8,77$

#### Load combinations

A possible collapse of the double tapered beam is of such nature that it might involve a high risk of injury to persons. Therefore, the safety class is assumed to be high (safety class 3), thus  $\gamma_d = 1$ . The beams are assumed to be indoors, in a heated environment. Therefore, they are characterized by an environment in which relative humidity very seldom, if ever, exceeds 65 %. Thus, the service class can be assumed to be 1.

Table 3.2

Safety class	Service class	Load combinations [kN/m]	Load duration	$K_{mod}$	$K_{def}$
<b>Serviceability limit state (SLS)</b>					
–	1	$g_k = (g_{1k} + g_{1k}) = 5,1$		–	0,6
–	1	$s = 8,77$		–	0,6
<b>Ultimate limit state (ULS)</b>					
$3 \rightarrow \gamma_d = 1$	1	$g_d = 1 \cdot 1,2 \cdot (g_{1k} + g_{2k}) = 6,1$	permanent	0,6	–
$3 \rightarrow \gamma_d = 1$	1	$q_d = 1 \cdot [1,2 \cdot (g_{1k} + g_{2k}) + 1,5 \cdot s] = 19,23$	medium-term	0,8	–

## Material

The material used for this structure is glulam GL30c ( $\gamma_M = 1,25$ ,  $k_{mod} = 0,8$ ).

Table 3.3

Characteristics	Design values
Bending	$f_{m,d} = 19,2$ MPa
Shear	$f_{v,d} = 2,2$ MPa
Compression parallel to grain	$f_{c,0,d} = 15,7$ MPa
Compression perpendicular to grain	$f_{c,90,d} = 1,6$ MPa
Tension parallel to grain	$f_{t,0,d} = 12,5$ MPa
Tension perpendicular to grain	$f_{t,90,d} = 0,32$ MPa
Modulus of elasticity	$E_{0,mean} = 13\ 000$ MPa
Shear modulus	$G_{mean} = 650$ MPa

### 3.5.1 Bending at critical section ( $x = x_0$ ) and at mid-span ( $x = \ell/2$ )

#### a. Determination of stresses

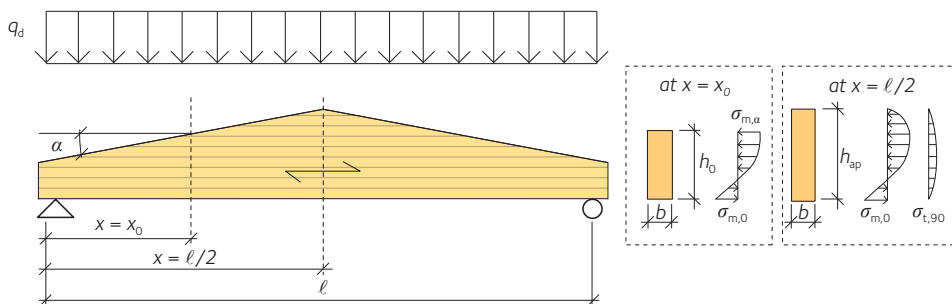


Figure 3.4

For a symmetrical double tapered beam with constant uniformly distributed load  $q_d$ , the location of the critical cross section – that is the abscissa where the maximum bending stress occurs – can be calculated as follows:

$$x_0 = \frac{\ell \cdot h}{2 \cdot h_{ap}} = \frac{20000 \cdot 700}{2 \cdot 1698} = 4122 \text{ mm}$$

The corresponding depth of the beam is:

$$h_0 = h + x_0 \cdot \tan 5,7^\circ = 700 + 4122 \cdot \tan 5,7^\circ = 1111 \text{ mm}$$

The bending moment at the critical cross section is:

$$M_0 = \frac{q_d \cdot x_0}{2} \cdot (\ell - x_0) = \frac{19,23 \cdot 4,12}{2} \cdot (20 - 4,12) = 629,3 \text{ kNm}$$

The corresponding bending stress at the critical cross section is:

$$\sigma_{m,\alpha,d} = \sigma_{m,0,d} = \frac{6 \cdot M_0}{b \cdot h_0^2} = \frac{6 \cdot 629,3 \cdot 10^6}{190 \cdot 1111^2} = 16,1 \text{ MPa}$$

The bending moment at mid-span is:

$$M_{ap,d} = \frac{q_d \cdot \ell^2}{8} = \frac{19,23 \cdot 20^2}{8} = 961,5 \text{ kNm}$$

The corresponding bending stress at mid span:

$$\sigma_{m,d} = \frac{6 \cdot M_{ap,d}}{b \cdot h_{ap}^2} = \frac{6 \cdot 961,5 \cdot 10^6}{190 \cdot 1698^2} = 10,5 \text{ MPa}$$

At the mid-span (apex zone), the tensile bending stress shall be magnified by a factor  $k_\ell$  to take into account of the fact that the depth of the beam is not constant, but it varies linearly and it has a singularity at the apex. The magnification factor  $k_\ell$  increases with increasing roof slope and it can be obtained from Volume 2: Section 8.2 or Eurocode 5: Equation 6.43. For a slope  $\alpha = 5,7^\circ \rightarrow k_\ell = 1,2$ .

$$\sigma_{m,d} = k_\ell \cdot \sigma_{m,d} = 1,2 \cdot 10,5 = 12,6 \text{ MPa}$$

## b. Verifications

At the tapered edge of the beam the bending strength must be reduced by a factor  $k_{m,\alpha}$  to take into consideration the effect of simultaneous action of compression parallel to grain, tension perpendicular to grain and shear. The reduction factor  $k_{m,\alpha}$  increases with increasing roof slope and it can be obtained from Volume 1: Figure 3.35. For a slope  $\alpha = 5,7^\circ \rightarrow k_{m,\alpha} = 0,86$ .

Table 3.4

Position	$\sigma_{m,d}$ [MPa]	$f_{m,d}$ [MPa]	$k_{m,\alpha}$	$f_{m,\alpha,d}$ [MPa]	Utilization ratio
$x = x_0$	16,1	19,2	0,86	$19,2 \cdot 0,86 = 16,5$	$16,1 / 16,5 = 0,97$
$x = \ell / 2$	12,6	19,2	–	–	$12,6 / 19,2 = 0,65$

$\sigma_{m,d}$  = design bending stress;  $f_{m,d}$  = design bending strength;  $f_{m,\alpha,d}$  = reduced design bending strength (tapered edge).

### Lateral torsional buckling

Lateral torsional buckling in the double tapered beam (primary beams) may only occur between two adjacent purlins, under the condition that 1) the roof is braced in the transverse direction and 2) the purlins are adequately fastened to the primary beams. In that case, the buckling length can be assumed as the centre-to-centre distance between purlins, that is  $\ell_2 = 2\,400$  mm. Within this distance, the depth of the cross section can be regarded as constant. The critical bending stress can be calculated according to Volume 1: Equation 3.34:

$$\sigma_{m,crit} \approx \frac{0,78 \cdot b^2}{h_0 \cdot \ell_{ef}} \cdot E_{0,05} = \frac{0,78 \cdot 190^2}{1111 \cdot 2400} \cdot 10800 = 114 \text{ MPa}$$

The relative slenderness ratio for bending is defined in Volume 1: Equation 3.30 and the corresponding reduction factor  $k_{crit}$  is defined in Volume 1: Table 3.3:

$$\lambda_{rel,m} = \sqrt{\frac{f_{m,k}}{\sigma_{m,crit}}} = \sqrt{\frac{30}{114}} = 0,51 < 0,75 \rightarrow k_{crit} = 1$$

- Since  $\lambda_{rel,m} \leq 0,75$ , full bending strength can be achieved without risk for lateral torsional buckling.

It is important to check the shear capacity for tapered beams due to the normally low beam depth at the supports. Yet this check is omitted in this example.

## 3.5.2 Tension perpendicular to the grain

### a. Determination of stresses

The tensile stress perpendicular to the grain can be evaluated by multiplying the bending stress at mid-span by the factor  $k_p$ , which can be taken from Volume 1: Figure 3.38:

$$\sigma_{t,90,d} = k_p \cdot \sigma_{m,d} = 0,02 \cdot 10,5 = 0,21 \text{ MPa}$$

### b. Verifications

The tension strength perpendicular to grain shall be reduced in order to take into account the volume effect. The volume of wood which is loaded in tension can be estimated as follows, see Volume 1: Table 3.4:

$$V = b \cdot h_{ap}^2 = 190 \cdot 1698^2 \cdot 10^{-9} = 0,55 \text{ m}^3$$

The reduction factor due to volume effect can be calculated according to Volume 1: Equation 3.53:

$$k_{vol} = k_{dis} \cdot \left(\frac{0,01}{V}\right)^{0,2} = 1,4 \cdot \left(\frac{0,01}{0,55}\right)^{0,2} = 0,63$$

where  $k_{dis}$  is a factor that takes into account that the tension stress perpendicular to the grain is not uniformly distributed in the loaded timber volume  $V$ .

Table 3.5

Position	$\sigma_{t,90}$ [MPa]	$f_{t,90,d}$ [MPa]	$k_{vol}$	$f_{t,90,d,red}$ [MPa]	Utilization ratio
$x = \ell/2$	0,21	0,32	0,63	$0,63 \cdot 0,32 = 0,20$	$0,21/0,20 \approx 1,045$

Here the need for reinforcement is just about necessary. A calculation with unequal-sided snowload according to EKS 10 results in a reduction of the bending moment to 97 % compared to the one in the calculation above. The utilization ratio is then  $0,97 \cdot 1,045 = 1,01$ , what could be acceptable.

Additionally the shear capacity at the supports has to be verified. This is of special importance for double tapered beams because of their reduced cross sectional height at the supports. By slopes 1/10 or more the shear stresses at the supports are often designing the beam dimensions.

### 3.5.3 Shear at the supports

#### Support reaction

$$R = 19,229 \cdot 20 / 2 = 192,293 \text{ kN}$$

Advantageously the shear force is given by applying the possible reduction according to Eurocode 5: Section 6.1.7 (3). With purlins spaced at 2400 mm centres a load on 1 200 mm of the primary beam goes directly to the support and does so not contribute to the shear stress.

$$V = R - 1,2 \cdot 19,229 = 169,218 \text{ kN}$$

$$\tau = 1,5 \cdot 169,218 \cdot 1000 / (0,19 \cdot 0,7) = 1,908 \cdot 10^6 \text{ N/m}^2$$

$$f_{v,d} = 3,0 / 3,5 \cdot 3,5 \cdot 10^6 \cdot 0,8 / 1,25 = 1,920 \cdot 10^6 \text{ N/m}^2$$

The utilization ratio is then  $1,908 / 1,92 = 0,99$ , what is acceptable.

### 3.5.4 Compression perpendicular to the grain at the supports

#### a. Determination of stresses

We assume that the column supporting the double tapered beam has a cross section  $190 \times 360 \text{ mm}$ . The support length between beam and column is therefore  $b \times \ell_b = 190 \times 360 \text{ mm}^2$ .

The compression stress perpendicular to the grain can be evaluated according to Volume 2: Section 5.2 or Eurocode 5: Section 6.1.5.

$$\sigma_{c,90,d} = \frac{0,5 \cdot q_d \cdot \ell}{A_{ef}} = \frac{q_d \cdot \ell}{2 \cdot b \cdot (\ell_b + 30 \text{ mm})} = \frac{19,23 \cdot 20000}{2} \cdot \frac{1}{190 \cdot (360 + 30)} = 2,60 \text{ MPa}$$

#### b. Verifications

The compression strength perpendicular to grain can be magnified by factor  $k_{c,90} = 1,75$ . Because  $g_k/q_k > 0,4$  it is not recommended to choose  $k_{mod}$  and  $\gamma_M$  as 1,0.

Table 3.6

Position	$\sigma_{c,90,d}$ [MPa]	$f_{c,90,d}$ [MPa]	$k_{c,90}$	$f'_{c,90,d}$ [MPa]	Utilization ratio
$x = 0$	2,6	1,6	1,75	$1,75 \cdot 1,6 = 2,8$	$2,6 / 2,8 = 0,93$

### 3.5.5 Deflection

For a double tapered beam, resting on supports and subjected to a uniformly distributed load  $q$ , the deflection  $w$  should be evaluated by the following equation, Piazza et al. (2005), see Volume 1: Section 3.6:

$$w = \frac{5}{384} \cdot \frac{q \cdot \ell^4}{E_{0,\text{mean}} \cdot I_y} \cdot k_m + \chi \cdot \frac{1}{8} \cdot \frac{q \cdot \ell^2}{G_{\text{mean}} \cdot A} \cdot k_v$$

where  $\chi = 1,2$ ,  $I_y$  and  $A$  are the moment of inertia and area of the cross section at the support respectively, whilst  $k_m$  and  $k_v$  are defined as:

$$k_m = \left( \frac{h}{H} \right)^3 \cdot \frac{1}{0,15 + 0,85 \cdot h/H} = \left( \frac{700}{1698} \right)^3 \cdot \frac{1}{0,15 + 0,85 \cdot 700/1698} = 0,14$$

$$k_v = \frac{2}{1 + (H/h)^{2/3}} = \frac{2}{1 + (1698/700)^{2/3}} = 0,71$$

The instantaneous deflections are:

- $w_{g,\text{inst}}$  – due to self weight
- $w_{qs1,\text{inst}}$  – due to the variable action  $q_{s1}$  (snow load)

$$w_{g,\text{inst}} = \frac{5}{384} \cdot \frac{g_k \cdot \ell^4}{E_{0,\text{mean}} \cdot I_y} \cdot k_m + \chi \cdot \frac{1}{8} \cdot \frac{g_k \cdot \ell^2}{G_{\text{mean}} \cdot A} \cdot k_v = \frac{5}{384} \cdot \frac{5,1 \cdot 20000^4 \cdot 12}{13000 \cdot 190 \cdot 700^3} \cdot 0,14 + 1,2 \cdot \frac{1}{8} \cdot \frac{5,1 \cdot 20000^2}{650 \cdot 190 \cdot 700} \cdot 0,71 = 23 \text{ mm}$$

$$w_{qs1,\text{inst}} = \frac{5}{384} \cdot \frac{s \cdot \ell^4}{E_{0,\text{mean}} \cdot I_y} \cdot k_m + \chi \cdot \frac{1}{8} \cdot \frac{s \cdot \ell^2}{G_{\text{mean}} \cdot A} \cdot k_v = \frac{5}{384} \cdot \frac{8,77 \cdot 20000^4 \cdot 12}{13000 \cdot 190 \cdot 700^3} \cdot 0,14 + 1,2 \cdot \frac{1}{8} \cdot \frac{8,77 \cdot 20000^2}{650 \cdot 190 \cdot 700} \cdot 0,71 = 41 \text{ mm}$$

With  $\Psi_{2,1} = 0,1$  (snow load) and  $k_{\text{def}} = 0,6$  (service class 1), the final deflection is:

$$w_{\text{fin}} = w_{g,\text{inst}} \cdot (1 + k_{\text{def}}) + w_{qs1,\text{inst}} \cdot (1 + \Psi_{2,1} \cdot k_{\text{def}}) = 23 \cdot (1 + 0,6) + 41 \cdot (1 + 0,1 \cdot 0,6) = 37 + 43 = 80 \text{ mm}$$

► **This deflection corresponds to  $\ell/249$  which is fully acceptable for an industrial building.**

For schools, stores and similar facilities with higher requirements, it can be considered to manufacture the beam with a precamber of say 70 mm.

## 4 Design of timber joints

### Example 4.1 Design of tie rod connection

Hint: Failure mode (d) in Volume 2: Figure 10.2 is governing the behaviour of the connection and the rope effect is not necessary to take into account.

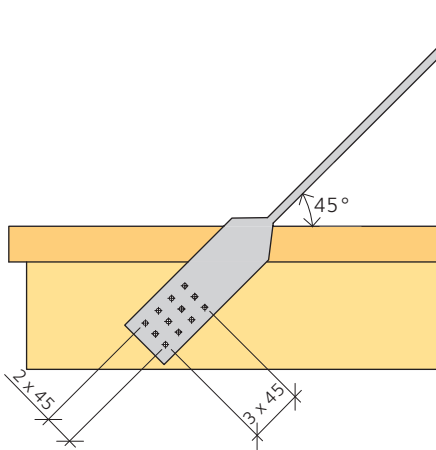


Figure 4.1

A balcony in timber is at the front edge supported by a tie rod made of steel. This steel rod is attached to the floor beam of the balcony with 12 wood screws. The supporting floor beam has the dimension  $115 \times 270$  mm. The wood screws have a length of 60 mm, a diameter of 8 mm and an ultimate strength of 410 MPa. For simplicity the effective diameter  $d_{ef}$  is here assumed to be equal the outer thread diameter  $d$ . This information is usually to be found in the declarations from the screw manufacturers. The thickness of the steel plate is 8 mm. The tensile resistance perpendicular to the grain of the floor beam is assumed to be sufficient to withstand the force from the tie rod.

**Calculate the maximum load in the tie rod with respect to the capacity of the steel-to-wood connection.**

Spacings and edge distances for the screws are assumed to be adequate.

#### Geometry

Steel thickness	$t = 8$ mm
Screw diameter	$d_{ef} = d = 8$ mm
Screw length	$\ell_s = 60$ mm
Screw length in the timber	$t_1 = \ell_s - t = 52$ mm
Angle between force and grain direction	$\alpha = 45^\circ$
Number of wood screws	$n = 12$



**Capacity of the connection**

Characteristic density, glulam GL30c	$\rho_k = 390 \text{ kg/m}^3$
Wood screw strength	$f_u = 410 \text{ MPa}$
Partial coefficient, connection	$\gamma_M = 1,3$
Material factor for the connection at the ULS, medium term load M, service class 3	$k_{\text{mod}} = 0,65$
Yield moment wood screw	$M_{y,Rk} = 0,3 f_u d_{\text{ef}}^{2,6}$ $M_{y,Rk} = 2,74 \cdot 10^4 \text{ Nmm}$
Embedding strength parallel fibers	$f_{h,0,k} = 0,082(1 - 0,01d_{\text{ef}}) \rho_k$ $f_{h,0,k} = 29,42 \text{ MPa}$
Softwood, correction factor	$k_{90} = 1,35 + 0,015d_{\text{ef}} = 1,47$
Embedding strength in the direction of the force	$f_{h,\alpha,k} = \frac{f_{h,0,k}}{k_{90} \sin^2 \alpha + \cos^2 \alpha}$ $f_{h,\alpha,k} = 23,82 \text{ MPa}$

The connection is a one sided connection with a thick steel plate, that is failure mode c, d or e, see Volume 2: Figure 10.2 and Volume 1: Table 4.3:

Failure mode c	$F_{v,Rk,c} = \dots = 9,91 \text{ kN}$
Failure mode d	$F_{v,Rk,d} = \dots = 4,83 \text{ kN}$
Failure mode e	$F_{v,Rk,e} = \dots = 5,26 \text{ kN}$
Characteristic capacity	$F_{v,Rk} = \min(F_{v,Rk,c}, F_{v,Rk,d}, F_{v,Rk,e}) = 4,83 \text{ kN}$
Design capacity	$F_{v,Rd} = \dots = 2,42 \text{ kN}$
Total capacity of the connection	$F_v = n \cdot F_{v,Rd} = 29,0 \text{ kN}$

► **The connection can withstand a force of 29 kN.**

## Example 4.2 Design of nailed connection in a Gerber system

This connection can advantageously be designed so that the force is being transferred via compression towards the horizontal parts of the steel, what also is the common procedure in practice. However in the example this have been ignored and focus is to show how nail design is done.

### 4.2.1 Problem description

The Gerber connection is placed in the outer bay of a beam with a span in the bay of 23 000 mm. The beam is loaded with an evenly distributed load.

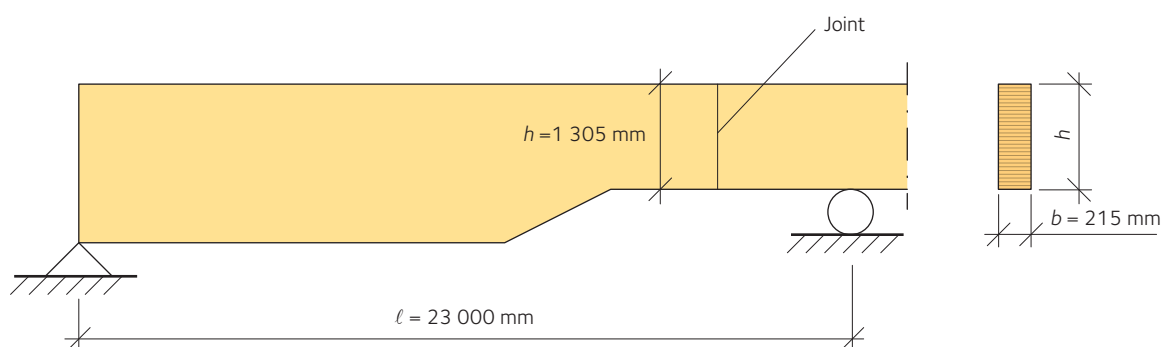


Figure 4.2: Geometry.

- $q_d = 34,3$  kN/m
- Load duration class M
- Nail material,  $f_u = 600$  MPa
- Material GL30c,  $b \times h = 215 \times 1\,305$  mm

The shear force to be transferred in the connection is calculated as:

$$V = 0,44q_d \ell = 0,44 \cdot 34,3 \cdot 23 = 347 \text{ kN}$$

### 4.2.2 Approach

The Gerber connection is always placed where the moment is (theoretically) zero, which means that the shear force is relatively large. The connection shall thus be designed so that no moment is transferred. A principal sketch is shown in Figure 4.3.

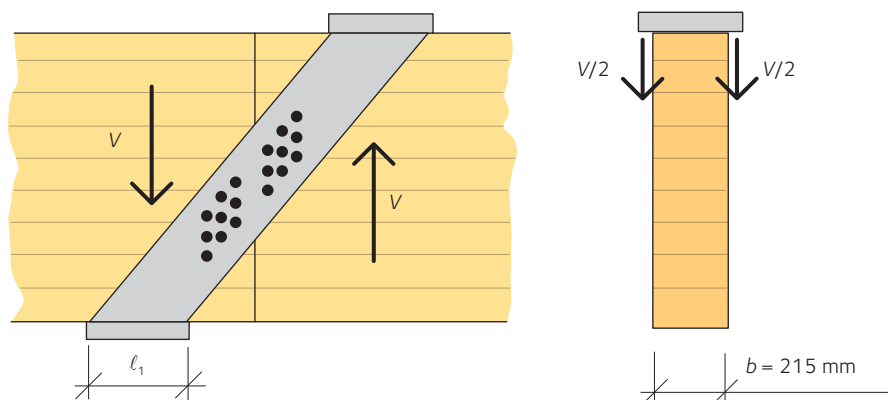


Figure 4.3: Principal solution for Gerber connection.

In smaller Gerber connections it is possible to use a simple rectangular nail plate placed centrally in the beam. But for the large connection in Figure 4.3, the steel must also be designed. Please refer to Eurocode 3 for further details on steel design.

### 4.2.3 Resistance of nails

Annular ring shanked nails of the dimension 40–4,0 mm are assumed. They have a head diameter of 6 mm. The connection will be a single shear connection steel-wood. The thickness of the steel plate is chosen as 5 mm, which means that  $t_{\text{steel}} > d$  ( $5 > 4$ ) and the nail has a fixed support in the steel plate. Volume 1: Table 4.3 contains the three failure modes that need to be calculated. The lowest of these three values will be the characteristic resistance for one nail. The embedding strength is established through using  $\rho_k = 390 \text{ kg/m}^3$  in Volume 1: Equation 4.1.

$$f_{h,k} = 0,082 \rho_k d^{-0,3} = 0,082 \cdot 390 \cdot 4^{-0,3} = 21,1 \text{ MPa}$$

The yield moment of the nail  $M_{y,Rk}$  with  $d = 4 \text{ mm}$ :

$$M_{y,Rk} = 600/600 \cdot 270 \cdot 4^{2,6} = 9\,925 \text{ Nmm}$$

Somewhat simplified the formula for smooth, grooved nails has here been applied. In practice this value is usually obtained from each nail manufacturer.

#### 4.2.3.1 Shear capacity

The thickness  $t_1$  is the length of the nail minus the thickness of the steel plate;  $t_1 = 40 - 5 = 35 \text{ mm}$ :

$$F_{v,Rk} \text{ (mode I)} = f_{h,k} t_1 d = 21,1 \cdot 35 \cdot 4 = 2\,954 \text{ N}$$

$$F_{v,Rk} \text{ (mode II)} = 21,1 \cdot 35 \cdot 4 \cdot (\sqrt{(2 + 4 \cdot 9925 / (21,1 \cdot 4 \cdot 35^2))} - 1) = 1\,607 \text{ N}$$

$$F_{v,Rk} \text{ (mode III)} = 2,3 \cdot \sqrt{(9925 \cdot 21,1 \cdot 4)} = 2\,105 \text{ N}$$

The lowest capacity of the three failure modes occurs for failure mode II, that is  $F_{v,Rk} = 1,607 \text{ kN/nail}$ . The design value is determined using Eurocode 5 with  $k_{\text{mod}} = 0,8$  and  $\gamma_M = 1,3$  (also see Volume 2: Section 3.1 and 3.2):

$$F_{v,Rd} = 0,8 \cdot 1,607 / 1,3 = 0,989 \text{ kN/nail}$$

#### 4.2.3.2 Axial capacity

The axial capacity is determined using Volume 1: Equations 4.14–4.16 and  $t_{\text{pen}} = t_1 = 35 \text{ mm}$ :

$$f_{ax,k} = 20 \cdot 10^{-6} \cdot 390^2 = 3,04 \text{ MPa} \Rightarrow F_{ax,Rk} = 3,04 \cdot 4 \cdot 35 = 426 \text{ N}$$

$$f_{\text{head},k} = 70 \cdot 10^{-6} \cdot 390^2 = 10,6 \text{ MPa} \Rightarrow F_{ax,Rk} = 10,6 \cdot 6^2 = 383 \text{ N}$$

$$F_{ax,Rk} = \min(426; 383) = 383 \text{ N} = 0,383 \text{ kN/nail}$$

In practice also these values are usually obtained from each nail manufacturer.

### 4.2.3.3 Fastener capacity

The fastener capacity is obtained by adding the shear and the axial capacities according to (where the second term in the right part is the rope effect):

$$F_{Rk} = 1,607 + 0,383/4 = 1,70 \text{ kN/nail}$$

The contribution from rope effect in relation to the shear capacity is only 6 percent and can therefore be completely counted, as the limit is 50 percent for annular ring shanked nails, see Volume 1: Table 4.8.

The design capacity is determined using  $k_{mod} = 0,8$  and  $\gamma_M = 1,3$ :

$$F_{Rd} = \frac{1,70 \cdot 0,8}{1,3} = 1,05 \text{ kN/nail}$$

### 4.2.4 Connection design

The number of nails needed on each side of the connection is:

$$n = \frac{347 / 2}{1,05} = 166 \text{ nails}$$

The nails should be placed as close to the neutral axis of the beams as possible to avoid eccentricity moments. The regulations for spacing parallel and perpendicular to grain needs to be followed, Volume 2: Section 10.4.2. For nailed joints, a distance of  $5d$  is prescribed perpendicular to the grain and  $10d$  parallel to the grain. A proposed placement of fasteners would be according to Figure 4.4:

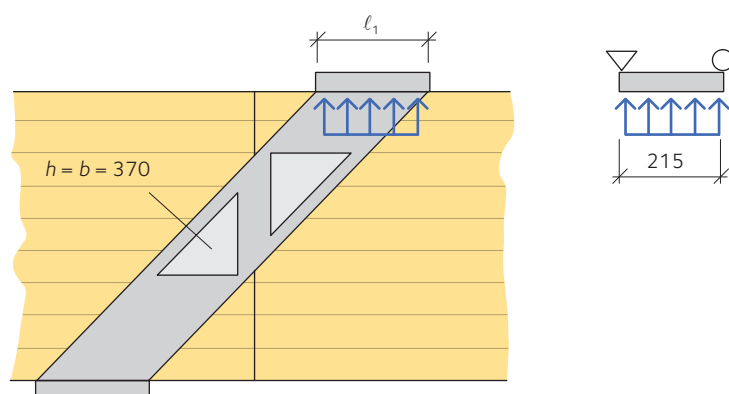


Figure 4.4

The size of the triangle where fasteners are to be placed is derived by assuring that 166 nails fit into the triangle without violating fastener spacing recommendations. To complete the design of the connection, the steel plates would need to be designed, refer to Eurocode 3 for steel design. Furthermore, the compression perpendicular to the grain of the wood must be checked as indicated in Figure 4.4, see Volume 2: Section 5.2.

### Example 4.3 Design of bolted connection in tension

#### 4.3.1 Problem description

A bolted connection in pure tension shall be designed. The connection could be the bottom chord in a truss or a purlin in tension. The number of bolts is unknown at the onset of design.

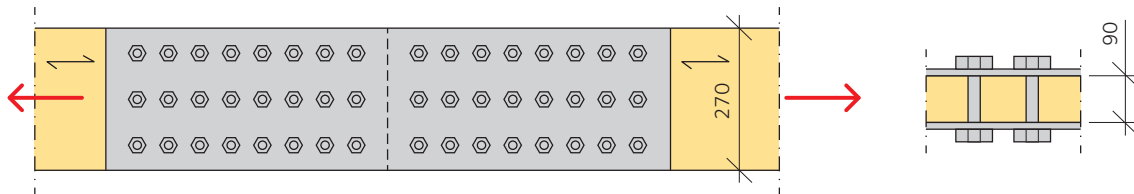


Figure 4.5

- design tensile force  $N_{td} = 250$  kN
- thickness of the steel plate  $t = 8$  mm
- load duration class M, service class 2
- bolt diameter  $d = 16$  mm
- tensile strength of bolt  $f_u = 360$  MPa
- glulam GL30c,  $\rho_k = 390$  kg/m<sup>3</sup>.

#### 4.3.2 Strength parameters

The yield moment of the bolt is calculated using Volume 1: Equation 4.6:

$$M_{y,Rk} = 0,3 f_u d^{2,6} = 0,3 \cdot 360 \cdot 16^{2,6} = 146 \text{ Nm}$$

The embedding strength of wood is determined according to Volume 1: Equation 4.2:

$$f_{h,0,k} = 0,082(1 - 0,01 \cdot 16) \cdot 390 = 26,9 \text{ MPa}$$

#### 4.3.3 Resistance for a single bolt

The resistance for a single bolt is the sum of the shear and tensile resistances.

##### 4.3.3.1 Shear capacity

The shear capacity is established through using the expressions in Volume 1: Table 4.7, double shear steel-to-timber joints with  $t = 0,5d$ , failure mode j and k, that is pinned support for the bolt in relation to the steel plate:

$$F_{v,Rk} = \min \left\{ \begin{array}{l} 0,5 f_{h,2,k} t_2 d = 0,5 \cdot 26,9 \cdot 10^6 \cdot 0,090 \cdot 0,016 = 19,3 \text{ kN} \\ 1,15 \sqrt{2 M_{y,Rk} f_{h,2,k} d} = 1,15 \sqrt{2 \cdot 146 \cdot 26,9 \cdot 10^6 \cdot 0,016} = 12,9 \text{ kN} \end{array} \right.$$

Thus, the characteristic shear capacity is 12,9 kN/bolt.

#### 4.3.3.2 Withdrawal capacity

The withdrawal capacity of a bolt is determined using Volume 1: Equation 4.18. Since a washer is not used, but rather a continuous steel plate, the fictitious washer diameter is  $\min(12 \cdot 8; 4 \cdot 16) = 64 \text{ mm}$ :

$$A_{\text{washer}} = \pi(64^2 - 16^2)/4 = 3\,016 \text{ mm}^2$$

$$F_{\text{ax, washer, Rk}} = 3 \cdot 2,5 \cdot 3\,016 = 22,6 \text{ kN/side}$$

#### 4.3.3.3 Bolt resistance

The resistance of one bolt is determined:

$$F_{\text{Rk}} = F_{\text{v, Rk}} + 2 \cdot F_{\text{ax, washer, Rk}}/4 = 12,9 + 2 \cdot 22,6/4 = 24,2 \text{ kN/bolt}$$

Because a continuous steel plate exists, the whole withdrawal capacity is counted (rope effect). The design capacity is determined using  $k_{\text{mod}} = 0,8$  and  $\gamma_{\text{M}} = 1,3$ :

$$F_{\text{Rd}} = \frac{0,8 \cdot 24,2}{1,3} = 14,9 \text{ kN/bolt}$$

#### 4.3.4 Connection layout

The number of bolts will depend on the group effect and the number of rows in the connection. The group effect is in turn dependent on the number  $n$  of fasteners in a row, and therefore it is of interest to determine the number of rows that can be placed within the beam height 270 mm. The minimum distance between fasteners in a row is  $4d$  according to Volume 2: Table 10.4. The edge distance to an unloaded edge is  $3d$ . Thus, the maximum number of rows that can be placed perpendicular to grain is:

$$n_{\text{rows}} = \left( \frac{270 - 2 \cdot 3d}{4d} \right) + 1 = 3 \text{ rows}$$

#### 4.3.5 Group effect

According to Volume 1: Equation 4.23, bolted joints are affected by a group effect. The spacing for bolts parallel to the grain is chosen as  $7d$ , which is more than what the standard requires. Knowing that the boundary for  $n$  to result as  $n_{\text{ef}}$  is a spacing of  $13d$ , the determination of the number of fasteners in one row can be established as:

$$3n_{\text{ef}} \cdot 14,9 = 250$$

$$n^{0,9} \sqrt[4]{\frac{7d}{13d}} = \frac{250}{3 \cdot 14,9}$$

$$n \geq 8,04 \approx 8 \text{ bolts/row}$$

Without the group effect the connection could have been established using  $250/14,9 = 17$  bolts, but now 24 bolts results. If more rows could be accommodated within the beam height, this would yield a lower number of bolts in total. Increasing the beam height can therefore be a good suggestion if not other parameters affect the selection of height.

For a connection of this type, a block shear check would have been necessary in practice, see Volume 1: Section 4.9.3.



### 1. Geometry

Column width	$b_p = 215 \text{ mm}$
Column depth	$h_p = 315 \text{ mm}$
Length steel sheet	$h = 650 \text{ mm}$
Width steel sheet	$b = 200 \text{ mm}$
Thickness steel sheet	$t = 5 \text{ mm}$
Diameter nail	$d = 4 \text{ mm}$
Diameter nail head	$d_h = 8 \text{ mm}$
Diameter hole drilled in the steel sheet	$d_{\text{hole}} = d + 1 \text{ mm} = 5 \text{ mm}$
Nail length	$\ell_s = 60 \text{ mm}$
Number of nails $\perp$ grain	$n_h = 6$
Number of nails // grain	$n_v = 10$
Penetration depth, nail	$t_1 \ t_{\text{pen}} = \ell_s - t = 55 \text{ mm}$

### 2. Material properties

Shear strength glulam GL30c	$f_{v,k} = 3,5 \text{ MPa}$
Tension strength parallel to the grain, glulam GL30c	$f_{t,0,k} = 19,5 \text{ MPa}$
Compression strength perpendicular to the grain, glulam GL30c	$f_{c,90,k} = 2,5 \text{ MPa}$
Density, glulam GL30c	$\rho_k = 390 \text{ kg/m}^3$
Tension strength, nail	$f_u = 600 \text{ MPa}$
Yield strength, S235	$f_{yk} = 235 \text{ MPa}$
Ultimate strength, S235	$f_{uk} = 340 \text{ MPa}$

### 3. Modification factors

Material factor for glulam at ULS, GL30c	$\gamma_M = 1,25$
Material factor for glulam at ULS, Connection	$\gamma_{M,\text{connection}} = 1,3$
Material factor for glulam at ULS, Steel	$\gamma_{M2} = 1,2$
Factor for medium duration loading and service class 2	$k_{\text{mod}} = 0,8$



#### 4. Design section forces

Horizontal force  $H = 25 \text{ kN}$

Vertical force  $V = 25 \text{ kN}$

Moment  $M = 25 \text{ kNm}$

The horizontal force is taken as contact pressure between the column and the steel sheet.  
The vertical force is taken as contact between the bottom of the column and the foundation.  
The moment is taken as a force couple in the steel sheets.

The resulting force due to the moment  $F_x = \frac{M}{h_p} = 79 \text{ kN}$

This assumes that the force is so small so that the steel sheet does not buckle, which has to be checked. In this case Eurocode 3: Section 6.3 gives us:

$$\lambda = 60/5 \cdot \sqrt{12}/93,9 = 0,443$$

$$\Phi = 0,5 \cdot (1 + 0,49 \cdot (\lambda - 0,2) + \lambda^2) = 0,657$$

$$X = 1/(\Phi + \sqrt{\Phi^2 - \lambda^2}) = 0,875$$

The capacity of the steel sheet with respect to buckling is then  $N_{b,Rd} = 0,875 \cdot 0,005 \cdot 0,2 \cdot 235 \cdot 103/1,1 = 187 \text{ kN}$ . This means that in this case there is no risk of buckling, but that is so because the moment being moderate in the example and the bottom row of nails are placed as close to the lower end as possible, with respect to end distance ( $15d$ ). If the steel sheet would buckle, the lower end of the glulam column would transfer the force to the concrete through contact pressure and we would get a much shorter lever to withstand the moment and therefore an increased force in the tensioned sheet.

Force taken as contact between the steel sheet and the column.

$$F_y = H = 25 \text{ kN}$$

#### 5. Embedding strength

Embedding strength glulam GL30c  
(Volume 1: Equation 4.1)

$$f_{h,k} = 0,082\rho_k d^{-0,3}$$

$$f_{h,k} = 21,1 \text{ MPa}$$

#### 6. Yield moment, nail

Yield moment, nail  
(Volume 1: Equation 4.5)

$$M_{y,Rk} = 0,45f_u d^{2,6}$$

Somewhat simplified the formula for smooth, grooved nails has here been applied. In practice this value is usually obtained from each nail manufacturer.

$$M_{y,Rk} = 9\,925 \text{ Nmm}$$

## 7. Axial withdrawal capacity

Axial withdrawal strength  
(Volume 1: Equation 4.14)

$$f_{ax,k} = 20 \cdot 10^{-6} \rho_k^2$$

$$f_{ax,k} = 3 \text{ MPa}$$

Pull through strength

The steel sheet prevents pull through of the nail head, therefore the pull through strength is not being calculated.

Withdrawal capacity  
(Volume 1: Equation 4.16)

$$F_{ax,Rk} = f_{ax,k} \cdot d \cdot t_{pen} = 669 \text{ N}$$

Withdrawal capacity/4

$$F_{ax,Rk}/4 = 167 \text{ N}$$

## 8. Shear capacity of the nails

Thick steelplate  
(Volume 2: Section 10.3  
and Volume 1: Table 4.3)

$$t/d = 1,25 \quad \text{the condition } \geq 1 \text{ is satisfied}$$

Failure mode c  
(Volume 2: Section 10.3  
and Volume 1: Table 4.3)

$$F_{v,Rk,c} = f_{h,k} t_1 d = 21,1 \cdot 55 \cdot 4 = 4,64 \text{ kN}$$

Failure mode d  
(Volume 2: Section 10.3  
and Volume 1: Table 4.3)

$$F_{v,Rk,d} = f_{h,k} t_1 d \left( \sqrt{2 + \frac{4M_{y,Rk}}{f_{h,k} d t_1^2}} - 1 \right)$$

$$F_{v,Rk,d} = 21,1 \cdot 55 \cdot 4 \left( \sqrt{2 + \frac{4 \cdot 9925}{21,1 \cdot 4 \cdot 55^2}} - 1 \right) = 2,17 \text{ kN}$$

Failure mode e  
(Volume 2: Section 10.3  
and Volume 1: Table 4.3)

$$F_{v,Rk,e} = 2,3 \sqrt{M_{y,Rk} f_{h,k} d}$$

$$F_{v,Rk,e} = 2,3 \sqrt{9925 \cdot 21,1 \cdot 4} = 2,11 \text{ kN}$$

Capacity for one nail

$$F_{v,Rk} = \min(F_{v,Rk,c}, F_{v,Rk,d}, F_{v,Rk,e})$$

Deciding failure mode

Failure mode e

Contribution from rope effect  
(can here be fully counted)

$$F_{ax,Rk}/4 = 0,167 \text{ kN}$$

Total design capacity  
per nail and shear plane

$$F_{Rd} = \frac{(F_{v,Rk} + F_{ax,Rk} / 4) \cdot k_{mod}}{\gamma_M} = \frac{(2,11 + 0,167) \cdot 0,8}{1,3} = 1,40 \text{ kN}$$

Least spacing // grain for nails to be effective (reduction is not needed) (Volume 2: Table 10.1)	$14d = 56 \text{ mm}$
Real distance between nails	$a_1 = 60 \text{ mm}$
Total capacity for the connection	$F_{\text{Rd,tot}} = n_v \cdot n_h \cdot F_{\text{Rd}} = 10 \cdot 6 \cdot 1,40 = 84 \text{ kN}$
Control of the capacity	$\frac{F_x}{F_{\text{Rd,tot}}} = \frac{79}{84} = 0,94$

**The capacity of the nails is enough.**

### 9. Nail spacing

Chosen spacing parallel to the grain	$a_1 = 60 \text{ mm}$
Chosen spacing perpendicular to the grain	$a_2 = 30 \text{ mm}$
Chosen distance to loaded end	$a_3 = 60 \text{ mm}$
Angle between force direction and grain	$\alpha = 0$
Required spacing parallel to the grain	$a_1 = (5 + 5 \cdot  \cos(0) ) \cdot d \cdot 0,7 = 28 \text{ mm}$
Required spacing perpendicular to the grain	$a_2 = 5d \cdot 0,7 = 14 \text{ mm}$
Required distance to loaded end	$a_{3,t} = (10 + 5 \cdot \cos(0)) \cdot d = 60 \text{ mm}$

► **The capacity of the nails is enough to withstand the forces.**

To perform a complete check of the connection it is also necessary to check plug shear failure, the strength of the steel sheets, and if the steel sheet can withstand the horizontal force as well as to check the connection between the steel sheets and the foundation.

## 5 Composite timber elements

### Example 5.1 Design of an OSB-webbed I-girder (ULS and SLS)

A composite beam with flanges in strength class C24 and a web panel of oriented strand board (OSB 3) is designed as shown below. Check all important resistances in the ultimate limit state (ULS), except crushing perpendicular to grain due to support reactions. Check also the maximum instantaneous and final deflections in the serviceability limit state (SLS). For the total load the requirements for the characteristic load combination are  $w_{inst} \leq \ell/300$  and  $w_{fin} \leq \ell/200$ .

**Note** that the beam is braced sideways such that lateral torsional buckling cannot occur.

**Note also** that full composite action can be assumed between the flange timber and the web panel. Transverse web stiffeners are used at both supports.

The girder is located at a place where safety class 2 and service class 2 can be assumed. The rest of the input data are given below:

#### User defined units

Flange width for each side	$b_f = 45 \text{ mm}$
Flange depth	$h_f = 70 \text{ mm}$
Web panel depth	$h_w = 500 \text{ mm}$
Web panel thickness	$b_w = 15,0 \text{ mm}$
Total span	$\ell = 7,0 \text{ m}$
Self weight of the roof, including the girder	$g_k = 1,35 \text{ kN/m}$
Snow zone 2,5, snow is a medium term load in Sweden	$s_k = 3,60 \text{ kN/m}$ (Is obtained for example when one-sided slope of $< 15^\circ$ , without drifted snow and with spacing at 1800 mm centres)

Factor for the quasi permanent snow load in Sweden  $\psi_2 = 0,2$

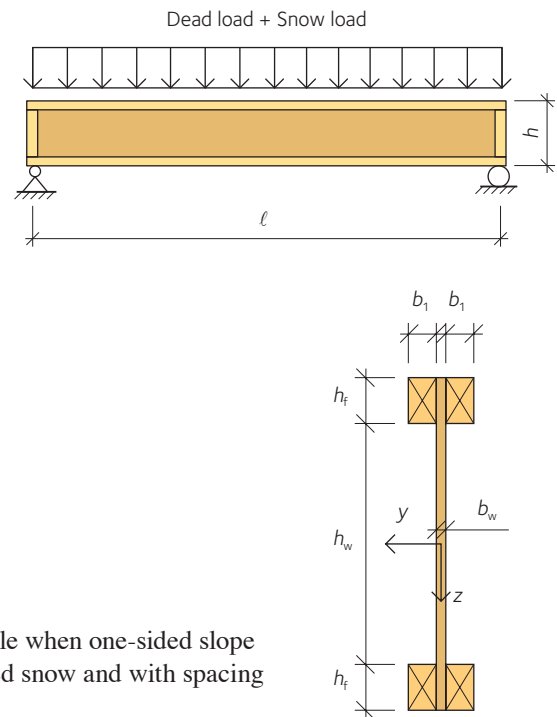


Figure 5.1

#### Material properties

Characteristic and design material properties of C24 timber (solid timber), see Volume 2, Table 3.3.

Bending strength

Tensile strength parallel to grain

Compressive strength parallel to grain

Shear strength

$$\left\{ \begin{array}{l} f_{m,k} \\ f_{t,0,k} \\ f_{c,0,k} \\ f_{v,k} \end{array} \right\} = \left\{ \begin{array}{l} 24 \\ 14,5 \\ 21 \\ 4,0 \end{array} \right\} \text{MPa}$$

Modulus of elasticity  
 $E_{0,mean} = 11\,000 \text{ MPa}$

Partial factor	$\gamma_M = 1,30$
Modification factor	$k_{mod} = 0,80$ (medium term load duration M and service class 2)
Creep factor	$k_{def} = 0,80$

The modification factor,  $k_h$  for size effects in bending and tension, is defined as:

$$k_{hhh}(h) = \begin{cases} k \leftarrow \min \left[ \left( \frac{150 \text{ mm}}{h} \right)^{0,2} \right] \\ 1,3 \\ k \leftarrow 1,0 \quad \text{if } k < 1,0 \\ k \end{cases}$$

For this flange we have:  $k_h = k_{hhh}(h_f) = 1,16$

Bending strength	$\begin{cases} \left( \begin{matrix} f_{m,d} \\ f_{t,0,d} \\ f_{c,0,d} \\ f_{v,d} \end{matrix} \right) = \frac{k_{mod}}{\gamma_M} \begin{pmatrix} f_{m,k} k_h \\ f_{t,0,k} k_h \\ f_{c,0,k} \\ f_{v,k} \end{pmatrix} = \begin{pmatrix} 1,72 \cdot 10^1 \\ 1,035 \cdot 10^1 \\ 1,29 \cdot 10^1 \\ 2,46 \cdot 10^0 \end{pmatrix} \text{ MPa} \end{cases}$
Tensile strength parallel to grain	
Compressive strength parallel to grain	
Shear strength	

**Characteristic and design material properties of OSB/3**, see Volume 2: Table 3.10.

In plane tensile strength	$\begin{cases} \left( \begin{matrix} f_{t,w,k} \\ f_{c,w,k} \\ f_{v,w,k} \\ f_{r,w,k} \end{matrix} \right) = \begin{pmatrix} 9,4 \\ 15,4 \\ 6,8 \\ 1,0 \end{pmatrix} \text{ MPa} \end{cases}$	Modulus of elasticity
In plane compressive strength		$E_{w,mean} = 3\,800 \text{ MPa}$
Panel shear strength		Shear modulus
Planar (rolling) shear strength		$G_{w,mean} = 1\,080 \text{ MPa}$

Partial factor for OSB/3	$\gamma_{M,OSB} = 1,2$
Modification factor for OSB/3	$k_{w,mod} = 0,55$ Medium term load duration and service class 2
Creep factor for OSB/3	$k_{w,def} = 2,25$

### Design values:

In plane tensile strength	$\begin{cases} \left( \begin{matrix} f_{t,w,d} \\ f_{c,w,d} \\ f_{v,w,d} \\ f_{r,w,d} \end{matrix} \right) = \frac{k_{w,mod}}{\gamma_{M,OSB}} \begin{pmatrix} f_{t,w,k} \\ f_{c,w,k} \\ f_{v,w,k} \\ f_{r,w,k} \end{pmatrix} = \begin{pmatrix} 4,31 \cdot 10^0 \\ 7,06 \cdot 10^0 \\ 3,12 \cdot 10^0 \\ 4,58 \cdot 10^{-1} \end{pmatrix} \text{ MPa} \end{cases}$
In plane compressive strength	
Panel shear strength	
Planar (rolling) shear strength	

### Design loads in the ULS, according to the Swedish annex of Eurocode 0

Partial factors	$\gamma_G = 1,35$	$\gamma_Q = 1,5$
Partial factor for safety class 2, Sweden	$\gamma_d = 0,91$	
$\xi$ -coefficient for permanent loads in Sweden	$\xi = 0,89$	

Total design load from combination, see Eurocode 0: Equation 6.10b, which is the only relevant combination due to a fairly large difference between  $s_k$  and  $g_k$ . Note that  $\gamma_d$  and  $\xi$  are from the Swedish application rules, EKS 10.

Design load  $q_{Ed} = \gamma_d (\xi \gamma_G g_k + \gamma_Q s_k) = 6,39 \text{ kN/m}$

### Cross sectional properties

As we have a cross-section of two different materials it is convenient to go for a transformed section or fictitious section as explained in Design of timber structures, Volume 1: Section 5.1.1.1. This cross-section will be different for the instantaneous, final SLS and final ULS conditions. Let the elastic modulus of the C24-flange be the reference material. The web thickness (parallel to the axis of bending) is then changed by the following factors:

Effective width factors

$$\left\{ \begin{array}{l} \mu_{w,SLS,inst} = \frac{E_{w,mean}}{E_{0,mean}} = 0,345 \\ \mu_{w,ULS,inst} = \mu_{w,SLS,inst} = 0,345 \\ \mu_{w,SLS,fin} = \frac{E_{w,mean}}{1 + k_{w,def}} \frac{(1 + k_{def})}{E_{0,mean}} = 0,191 \\ \mu_{w,ULS,fin} = \frac{E_{w,mean}}{1 + \psi_2 k_{w,def}} \frac{(1 + \psi_2 k_{def})}{E_{0,mean}} = 0,276 \end{array} \right.$$

Web thicknesses

$$\left\{ \begin{array}{l} b_{w,SLS,inst} = \mu_{w,SLS,inst} b_w = 5,18 \text{ mm} \\ b_{w,ULS,inst} = b_{w,SLS,inst} = 5,18 \text{ mm} \\ b_{w,SLS,fin} = \mu_{w,SLS,fin} b_w = 2,87 \text{ mm} \\ b_{w,ULS,fin} = \mu_{w,ULS,fin} b_w = 4,15 \text{ mm} \end{array} \right.$$

Total width of the timber flange  $b = 2b_1 = 90,0 \text{ mm}$

Total beam depth  $h = 2h_f + h_w = 640 \text{ mm}$

Function for the second area moment of inertia, which may be called for different web thicknesses

$$I_y(t_w) = \frac{(b + t_w)h^3 - b h_w^3}{12}$$

Second area moment of inertia of the transformed cross-sections

$$\begin{cases} I_{y,SLS,inst} = I_y(b_{w,SLS,inst}) = 1,14 \cdot 10^9 \text{ mm}^4 \\ I_{y,ULS,inst} = I_{y,SLS,inst} = 1,14 \cdot 10^9 \text{ mm}^4 \\ I_{y,SLS,fin} = I_y(b_{w,SLS,fin}) = 1,09 \cdot 10^9 \text{ mm}^4 \\ I_{y,ULS,fin} = I_y(b_{w,ULS,fin}) = 1,12 \cdot 10^9 \text{ mm}^4 \end{cases}$$

**Check the bending moment resistance**, see Volume 1: Section 5.1.1.2

No system effect can be taken into account, so  $k_{sys}$  is not used.

Also, no lateral torsional buckling can occur because we have sufficient bracing, that is  $k_{crit} = 1,0$

In this case the 3 most probable failure conditions are given to the right, which is realized by looking at the strength properties

$$\left( \begin{array}{l} \sigma_{f,t,d} = \frac{M_{y,Ed}}{I_{y,ULS,fin}} z_{flange,centre} \leq f_{t,0,d} \\ \sigma_{f,t,max,d} = \frac{M_{y,Ed}}{I_{y,ULS,fin}} z_{flange,edge} \leq f_{m,d} \\ \sigma_{w,t,max,d} = \mu_{w,ULS,inst} \frac{M_{y,Ed}}{I_{y,ULS,inst}} z_{flange,edge} \leq f_{t,w,d} \end{array} \right) \quad \text{(Volume 1: Equation 5.6)}$$

Just by looking at the numerical values of the parameters we understand that tension failure at the centre of the bottom flange will be the likely cause of failure. **Note** also that the flange is checked at final conditions, while the web edge is checked at instantaneous conditions. This is because the flanges have better creep properties than the web panel.

Maximum bending moment  $M_{y,Ed} = \frac{q_{Ed} \ell^2}{8} = 39,1 \text{ kNm}$

Stress in the C24-timber at the centre of the tension flange  $\sigma_{f,t,d} = \frac{M_{y,Ed}}{I_{y,ULS,fin}} \frac{h-h_f}{2} = 9,97 \text{ MPa}$   $f_{t,0,d} = 10,35 \text{ MPa}$

Stress in the C24-timber at the flange edge  $\sigma_{f,t,max,d} = \frac{M_{y,Ed}}{I_{y,ULS,fin}} \frac{h}{2} = 11,2 \text{ MPa}$   $f_{m,d} = 17,2 \text{ MPa}$

Stress in the OSB web panel at the edge of the tension flange  $\sigma_{w,t,max,d} = \mu_{w,ULS,inst} \frac{M_{y,Ed}}{I_{y,ULS,inst}} \frac{h}{2} = 3,79 \text{ MPa}$   $f_{t,w,d} = 4,31 \text{ MPa}$

- **We conclude that the tension flange is perfectly optimized with regard to bending failure of the timber flange as well as the web panel.**

**Check the shear force resistance at supports,** see Volume 1: Section 5.1.1.3

We have a pure shear failure of the web panel if the free web depth  $h_w = 500$  mm is smaller than  $35 b_w = 525$  mm. The smallest possible value of this limit is  $32 b_w = 480$  mm according to Volume 1: Table 5.2. Shear buckling will have a negligible effect on the shear resistance, which can be determined by Volume 1: Equation 5.11 with  $\chi_v = 1,0$ .

Maximum design shear force  $V_{z,Ed} = \frac{q_{Ed} \ell}{2} = 22,4$  kN

Shear force resistance without regarding shear buckling, noting that we have one web panel  $V_{z,Rd} = b_w (h_w + h_f) f_{v,w,d} = 24,6$  kN

*Compare!*

No fiddling around with fictitious cross sections is need, as the flanges are not involved in this failure. We conclude that also the web panel is well designed with regard to its shear resistance.

**Check for failure in the glue line between the web panel and the flange timber,** see Volume 1: Section 5.1.1.4

The failure criterion (Volume 1: Equation 5.20 or 5.21) is

$$\tau_{v,Ed} = \frac{V_{z,Ed} \Delta S}{I_y h_{gl}} \leq \begin{cases} f_{r,d} & \text{if } h_{gl} \leq \frac{4b_w}{n_{gl}} \\ f_{r,d} \left( \frac{4b_w}{n_{gl} h_{gl}} \right)^{0,8} & \text{if } h_{gl} > \frac{4b_w}{n_{gl}} \end{cases}$$

Here we have an I-beam for which  $n_{gl} = 2$ , that is 2 equal flange pieces are glued to both sides of a web panel. **Note** that  $b_w$  is the physical thickness and not a fictitious thickness. It is just that the shear stresses are more concentrated towards the inner corner in an I-beam than in a boxed beam that is taken into account through  $n_{gl}$ .

The depth of the glue line is  $h_{gl} = h_f = 70$  mm, compare to  $\frac{4b_w}{n_{gl}} = 30$  mm

We must, therefore, use a reduced planar (rolling) shear strength of the board material, which is lower than the shear strength of the timber.

$\Delta S$  is the first moment of area of the flange part belonging to each glue line, that is taken for the area that builds up the shear stress in the glue line, noting that the shear stress is zero at the free edge.

First area moment  $\Delta S_y = \frac{b_1}{2} h_f \frac{h - h_f}{2} = 4,49 \cdot 10^5$  mm<sup>3</sup>

In this case the  $I_{y,ULS,fin}$  will produce the largest shear stress. **Note** also that no  $\mu_{w,ULS,fin}$  is needed because the stress is already smeared out along the real physical depth of the glue line.



The shear stress is

$$\tau_{\text{mean,d}} = \frac{V_{z,\text{Ed}} \Delta S_y}{I_{y,\text{ULS,fin}} h_{\text{gl}}} = 0,128 \text{ MPa}$$

Compare to the reduced  
planar shear strength

$$f_{t,w,d} \left( \frac{4b_w}{n_{\text{gl}} h_{\text{gl}}} \right)^{0,8} = 0,233 \text{ MPa}$$

*Compare!*



► **We conclude that the glue line has more than sufficient strength.**

**Check the maximum beam deflection in the SLS,**

see Volume 1: Section 5.1.1.5

The deflection is to be calculated for the characteristic combination, that is as in Eurocode 0, Equation 6.14b.

$$\sum_{j \geq 1} G_{k,j} + Q_{k,1} + \sum_{i \geq 2} (\psi_{0,i} Q_{k,i})$$

Self weight

$$g_{\text{ser}} = g_k = 1,35 \text{ kN/m}$$

Snow load

$$s_{\text{ser}} = s_k = 3,60 \text{ kN/m}$$

Quasi permanent reduction factor for snow load, Sweden, snow zone 2,5

$$\psi_2 = 0,2$$

Creep factors for service class 2

<b>Timber</b>	<b>OSB</b>
$k_{\text{def}} = 0,8$	$k_{\text{w,def}} = 2,25$

Second area moments in the SLS

$$I_{y,\text{SLS,inst}} = 1,14 \cdot 10^9 \text{ mm}^4$$

$$I_{y,\text{SLS,fin}} = 1,09 \cdot 10^9 \text{ mm}^4$$

Area of the web panel, its real physical area

$$A_w = b_w h = 9,60 \cdot 10^3 \text{ mm}^2 \quad \text{if we are a bit brave, otherwise use the distance between flange centres}$$

Keep bending and shear deflections apart, because the web panels are much more prone to creep than the flanges acting in bending. The deflections calculated are based on Volume 1: Equations 5.85 and 5.86, but the “fca” index is replaced by “bend” for bending and “slip” is replaced by “shear”. The factor “2” is also removed.

Instantaneous bending deflection caused by the permanent load

\* Note that this is the instantaneous value to be used for the final bending deflection.

$$\left\{ \begin{array}{l} w_{g,\text{bend,SLS,inst}} = \frac{5g_{\text{ser}} \ell^4}{384 E_{0,\text{mean}} I_{y,\text{SLS,inst}}} = 3,36 \text{ mm} \\ w_{g,\text{bend,SLS,fin}} = \frac{5g_{\text{ser}} \ell^4}{384 E_{0,\text{mean}} I_{y,\text{SLS,fin}}} = 3,52 \text{ mm}^* \end{array} \right.$$

Instantaneous shear deflection caused by the permanent load

Note that there is no difference between the two values as they are not based on a fictitious cross section.

\* Note that this is the instantaneous value to be used for the final shear deflection.

$$\left\{ \begin{array}{l} w_{g,\text{shear,SLS,inst}} = \frac{g_{\text{ser}} \ell^2}{8 G_{\text{w,mean}} A_w} = 0,798 \text{ mm} \\ w_{g,\text{shear,SLS,fin}} = w_{g,\text{shear,SLS,inst}} = 0,798 \text{ mm}^* \end{array} \right.$$

Instantaneous bending deflection caused by the snow load

\* Note that this is the instantaneous value to be used for the final bending deflection.

$$\left\{ \begin{array}{l} w_{s,\text{bend,SLS,inst}} = \frac{5s_{\text{ser}} \ell^4}{384 E_{0,\text{mean}} I_{y,\text{SLS,inst}}} = 8,96 \text{ mm} \\ w_{s,\text{bend,SLS,infin}} = \frac{5s_{\text{ser}} \ell^4}{384 E_{0,\text{mean}} I_{y,\text{SLS,fin}}} = 9,38 \text{ mm}^* \end{array} \right.$$

Instantaneous shear deflection caused by the snow load

Note that there is no difference between the two values as they are not based on a fictitious cross section.

\* Note that this is the instantaneous value to be used for the final shear deflection.

$$\left\{ \begin{array}{l} w_{s,\text{shear,SLS,inst}} = \frac{s_{\text{ser}} \ell^2}{8 G_{w,\text{mean}} A_w} = 2,13 \text{ mm} \\ w_{s,\text{shear,SLS,infin}} = w_{s,\text{shear,SLS,inst}} = 2,13 \text{ mm}^* \end{array} \right.$$

Total instantaneous bending deflection

$$w_{\text{bend,inst}} = w_{g,\text{bend,SLS,inst}} + w_{s,\text{bend,SLS,inst}} = 12,3 \text{ mm}$$

Total instantaneous shear deflection

$$w_{\text{shear,inst}} = w_{g,\text{shear,SLS,inst}} + w_{s,\text{shear,SLS,inst}} = 2,92 \text{ mm}$$

Total instantaneous deflection

$$w_{\text{inst}} = w_{\text{bend,inst}} + w_{\text{shear,inst}} = 15,2 \text{ mm} \quad \text{compare to } \frac{\ell}{300} = 23,3 \text{ mm}$$

Final bending deflection from permanent loading

$$w_{g,\text{bend,fin}} = w_{g,\text{bend,SLS,infin}} (1 + k_{\text{def}}) = 6,33 \text{ mm}$$

Final shear deflection from permanent loading

$$w_{g,\text{shear,fin}} = w_{g,\text{shear,SLS,infin}} (1 + k_{w,\text{def}}) = 2,59 \text{ mm} \quad \text{Note the more severe creep factor}$$

Final bending deflection from snow loading

$$w_{s,\text{bend,fin}} = w_{s,\text{bend,SLS,infin}} (1 + \psi_2 k_{\text{def}}) = 10,9 \text{ mm}$$

Final shear deflection from snow loading

$$w_{s,\text{shear,fin}} = w_{s,\text{shear,SLS,infin}} (1 + \psi_2 k_{w,\text{def}}) = 3,08 \text{ mm} \quad \text{Note the more severe creep factor}$$

Total final bending deflection

$$w_{\text{bend,fin}} = w_{g,\text{bend,fin}} + w_{s,\text{bend,fin}} = 17,2 \text{ mm}$$

Total final shear deflection

$$w_{\text{shear,fin}} = w_{g,\text{shear,fin}} + w_{s,\text{shear,fin}} = 5,68 \text{ mm}$$

Total final deflection

$$w_{\text{fin}} = w_{\text{bend,fin}} + w_{\text{shear,fin}} = 22,9 \text{ mm} \quad \text{compare to } \frac{\ell}{200} = 35,0 \text{ mm}$$

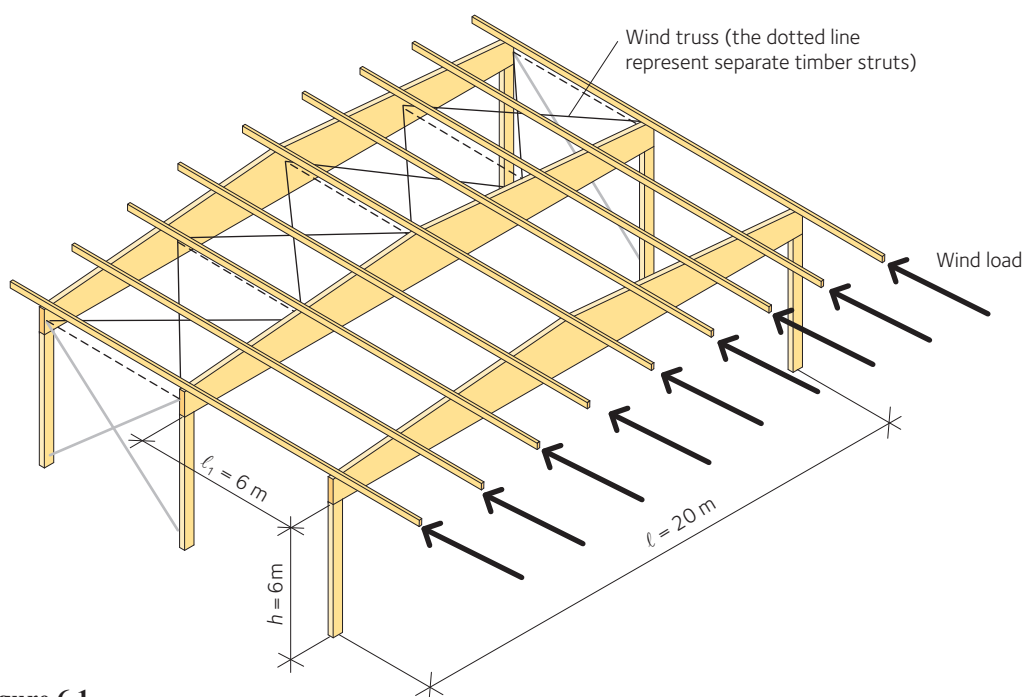
## 6 Horizontal stabilization

### **Example 6.1** Design of bracing system for wind load on the gable of an industrial hall

For missing data see Volume 3: Example 3.5 (this volume).

#### **Geometry of the structure:**

The building with the measures  $l \times l_{\text{tot}} 20 \times 36$  m is braced by means of a wind truss in the roof, consisting of diagonal steel bars and orthogonal compression struts of glulam. The wind truss is placed right below the roof plane. The horizontal loads are led by means of purlins to separate compression struts, which lay in the same plane as the diagonal steel bars. This to avoid eccentricities in the truss joints. The support reactions from the wind truss are led by means of a pair of diagonal steel bars in each longside wall to the foundation, but they are not designed in this example. The geometry of the structure is shown in Figure 6.1 below:

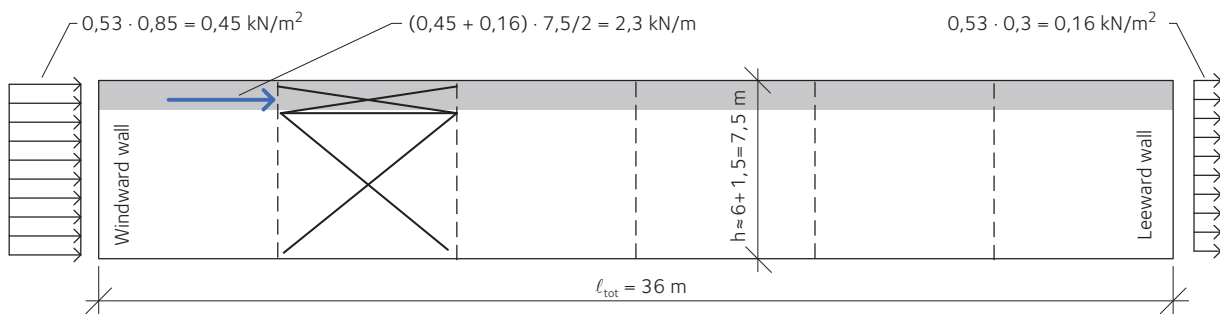


**Figure 6.1**

#### **Loads acting on the wind truss**

The loads considered for the design of the roof wind truss are: wind as primary action and second order effects generated by beam self weight, roof dead load and snow.

The characteristic value for wind load is assumed  $0,53 \text{ kN/m}^2$ , which generates a pressure at the windward wall and suction at the leeward wall. The pressure coefficient are assumed  $0,85$  and  $0,3$  for the windward wall and the leeward wall respectively. The wind loads considered for the design of the roof wind truss are shown in Figure 6.2 below.


**Figure 6.2**
**Table 6.1**

Load type	Uniformly distributed load [kN/m <sup>2</sup> ]	Uniformly distributed load [kN/m]	Load combination factor $\psi_0$
Wind	$0,45 + 0,16 = 0,61$	$w_k = 0,61 \cdot 7,5 / 2 = 2,3$	0,3
Beam self weight	–	$g_{1k} = 1,10$	–
Roof dead load	0,6	$g_{2k} = 3,96$	–
Snow	1,5	$s = 8,77$	0,6

In addition to wind, the load generated by the out-of-straightness of the main beams (second order effects) shall be taken into account. This load can be considered as uniformly distributed and its characteristic value can be calculated according to Equation 6.28:

$$q_h = n \cdot \frac{M}{30h \cdot \ell} \cdot (1 - k_{\text{crit}})$$

where:

- $n = 6$  is the number of braced beams (note that the total number of beams is 7, but the beams at the two gables carry only one half of the vertical load)
- $M$  is the bending moment in the beam due to vertical loads, considering the load combination “STR B”, according to Eurocode 0. The load combination factor should be taken  $\psi_0 = 0,6$  when wind is considered as principal load and  $\psi_0 = 1,0$  when snow is considered as principal load:

- $\ell = 20$  m 
$$M = \frac{q\ell^2}{8} = \frac{[1,2 \cdot (g_{1,k} + g_{2,k}) + 1,5 \cdot \psi_0 \cdot s] \cdot \ell^2}{8}$$

- $k_{\text{crit}}$  is the factor that takes into account the risk for lateral torsional buckling in an unbraced beam. The depth of the beam can be assumed  $h_m = (h + h_{\text{ap}})/2 \approx 1\,200$  mm and  $\ell_{\text{ef}} = 0,9 \cdot \ell + 2h$ :

$$\lambda_{\text{rel,m}} = \sqrt{\frac{f_{\text{m,k}}}{\sigma_{\text{m,crit}}}} = \sqrt{f_{\text{m,k}} \cdot \frac{h \cdot \ell_{\text{ef}}}{0,78 \cdot b^2 \cdot E_{0,05}}}$$

- $k_{\text{crit}}$  = see Volume 1: Table 3.3

In this example the tilting load from the columns is not considered.

Table 6.2

Load combination	Bending moment in the beam M [kNm]	$\lambda_{rel,m}$	$k_{crit}$	$q_h$ [kN/m]
a) Wind is the principal load	$\frac{[1,2 \cdot (1,1 + 3,96) + 0,6 \cdot 1,5 \cdot 8,77] \cdot 20^2}{8} = 698,5$	1,55	0,41	3,4
b) Snow is the principal load	$\frac{[1,2 \cdot (1,1 + 3,96) + 1,5 \cdot 8,77] \cdot 20^2}{8} = 961,5$	1,55	0,41	4,7

## Load combinations

The safety class is assumed to be high (safety class 3), thus  $\gamma_d = 1$ . The beams are assumed to be indoors, in a heated environment. Therefore, they are characterized by an environment in which relative humidity very seldom, if ever, exceeds 65 %. Thus, the service class can be assumed to be 1. In this example only load combination in the ULS is considered. Only the load combination with wind as principal load is shown in this example.

Table 6.3

Safety class	Service class	Load combinations [kN/m]	Load duration	$k_{mod}$
3 $\rightarrow \gamma_d = 1$	1	a) $q_{d,a} = 1 \cdot [1,5 \cdot w_k + 1,0 \cdot q_h] = 1,5 \cdot 2,3 + 3,4 = 6,9$	short	0,9
3 $\rightarrow \gamma_d = 1$	1	b) $q_{d,b} = 1 \cdot [1,0 \cdot q_h + 1,5 \cdot \psi_{0,w} w_k] = 1,0 \cdot 3,4 + 1,5 \cdot 0,3 \cdot 2,3 = 4,5$	medium	0,8

The governing load combination is thus a), that is wind as principal load.

## Material

### 1. Timber elements

The material used for the struts is glulam GL30c ( $\gamma_M = 1,25$ ,  $k_{mod} = 0,9$ ). The adopted cross section for the strut is  $190 \times 180$  mm. Upper chord and lower chord of the wind truss are two adjacent double tapered beams. The depth of these beams is varying along the span from 700 mm at support to 1 698 mm at mid-span. For the sake of the simplicity, however, it can be assumed that these beams have constant cross section  $190 \times 1\ 200$  mm<sup>2</sup>. The beams are also made of glulam GL30c. The relevant design values are shown below.

Table 6.4

	Design values
Compression parallel to grain	$f_{c,0,d} = 17,6$ MPa
Tension parallel to grain	$f_{t,0,d} = 14,0$ MPa
Modulus of elasticity	$E_{0,mean} = 13\ 000$ MPa $E_{0,05} = 10\ 800$ MPa

### 2. Steel diagonals

The diagonals are made of steel bars quality 4.8. The bars are threaded at their ends.

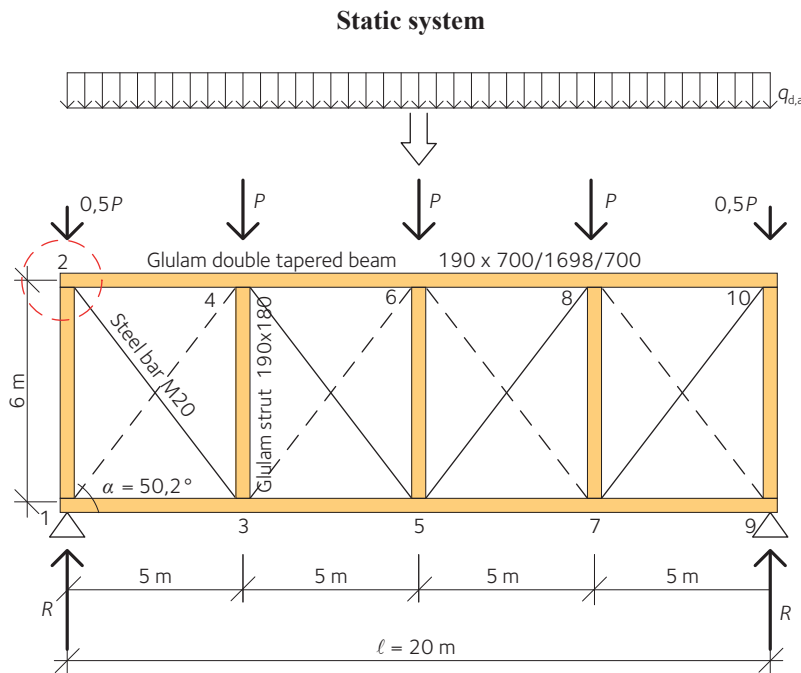
Table 6.5

Type of bar	Design yield stress $f_{yb}$ [MPa]	Design ultimate stress $f_{ub}$ [MPa]
Steel 4.8	320	400

### Design of the wind truss

The wind truss consists of a:

- lower chord and an upper chord of glulam
- orthogonal compression struts of glulam and
- diagonal bars of steel.



### Forces acting on the system

$$P = \frac{q_{d,a} \cdot \ell}{4} = \frac{6,9 \cdot 20}{4} = 34,5 \text{ kN}$$

$$R = \frac{4 \cdot P}{2} = \frac{4 \cdot 34,5}{2} = 69,0 \text{ kN}$$

Figure 6.3

#### a. Compression struts

The most loaded struts are those at the supports (that is struts 1 – 2 and 9 – 10). The force acting in these struts is:

$$N_{sd} = N_{1-2} = N_{9-10} = R = 69,0 \text{ kN}$$

Table 6.6

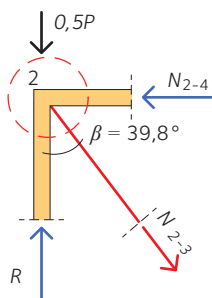
Cross section	Relative slenderness $\lambda_{rel} = \frac{\lambda}{\pi} \sqrt{\frac{f_{ck}}{E_{0,05}}}$	Reduction factor $k_c$	Compression force in the strut $N_{sd}$ [kN]	Compression strength of the strut $N_{Rd}$ [kN]	Utilization ratio
190 × 180	1,695	0,348	69,0	$k_c \cdot A \cdot f_{cd} =$ $0,348 \cdot 0,19 \cdot 0,18 \cdot 17,64 \cdot 10^3 =$ 210 kN	69,0/210 = 0,33

The same cross section, that is 190 × 180 mm is adopted for all the struts of the wind truss.

### b. Diagonal steel bars

The most loaded steel diagonals are those closest to the supports (that is diagonals 2 – 3 and 7 – 10). The force acting in these diagonals is:

#### Graphical representation of the forces acting in node 2



#### Forces acting in the steel bar

$$N_{sd} = N_{2-3} = \left( R - \frac{P}{2} \right) \frac{1}{\cos \beta} =$$

$$= \left( 69,0 - \frac{34,5}{2} \right) \cdot \frac{1}{\cos 39,8^\circ} = 67,4 \text{ kN}$$

Figure 6.4

The force acting in the diagonal steel bar is  $N_{sd} = 67,4$  kN. The design of the diagonal steel bars is performed as if they were bolts subjected to tensile force, since the end of the bars are threaded. The partial coefficient for bolts is  $\gamma_{M2} = 1,2$ .

Table 6.7

Steel quality	Size	Stress area $A_s$ [mm <sup>2</sup> ]	Ultimate stress $f_{ub}$ [MPa]	Tensile strength of the bar $F_{t,Rd} = 0,9 \cdot \frac{f_{ub} \cdot A_s}{\gamma_{M2}}$ [kN]
4.8	M12	84	400	25,2
4.8	M16	157	400	47,1
4.8	M20	245	400	73,5
4.8	M24	353	400	105,9

Steel bar M20, 4.8 are chosen as diagonal members in the wind truss.

►  $N_{sd} = 67,4 \text{ kN} < F_{t,Rd} = 73,5 \text{ kN} \rightarrow \text{OK}$



## 7 Design for serviceability

### Example 7.1 Design of a straight timber joist with respect to deflection

A simply supported rectangular joist is subjected to characteristic permanent loading,  $g_k = 0,5 \text{ kN/m}^2$  and characteristic medium duration variable load  $q_k = 2,0 \text{ kN/m}^2$ . The clear span  $\ell$  is 4,5 m and the joists are spaced at 0,6 m centres. The cross section is  $45 \times 220 \text{ mm}$ . Also see Example 3.1 in this Volume.

The joist is part of an indoor floor in a residential building. Service class 1 and safety class 2. Design the joist in structural timber of strength class C24.

Determine the deflection of the joist. Two different load cases will be discussed in the following to show different cases that might be of interest when checks of deflections are to be made.

#### Solution

First the deflection for characteristic load combination is determined, that is a deflection occurring quite seldom during the lifetime of the structure.

The moment of inertia  $I = 39,93 \cdot 10^{-6} \text{ m}^4$ .

Elastic modulus to be used in serviceability limit state  $E_{0,\text{mean}} = 11\,000 \text{ MPa}$

Characteristic load combination:

$$q_d = g_k + q_{k,1} + \sum \psi_{0,i} q_{k,i}$$

In this case  $g_k = 0,5 \text{ kN/m}^2$ , variable load is  $q_{k,1} = 2,0 \text{ kN/m}^2$  and there is no other variable load, that is:

$$\sum \psi_{0,i} q_{k,i} = 0$$

The deflection for a simple supported joist is:

$$w = \frac{5q\ell^4}{384EI}$$

The instantaneous deflection for the self-weight is:

$$w_{\text{inst,G}} = \frac{5g\ell^4}{384EI} = \frac{5 \cdot 0,5 \cdot 10^3 \cdot 0,6 \cdot 4,5^4}{384 \cdot 11000 \cdot 10^6 \cdot 39,93 \cdot 10^{-6}} \cdot 10^3 = 3,6 \text{ mm}$$

The instantaneous deflection for the variable load is:

$$w_{\text{inst,Q}} = \frac{5q\ell^4}{384EI} = \frac{5 \cdot 2,0 \cdot 10^3 \cdot 0,6 \cdot 4,5^4}{384 \cdot 11000 \cdot 10^6 \cdot 39,93 \cdot 10^{-6}} \cdot 10^3 = 14,6 \text{ mm}$$

In total the instantaneous (short term) deflection corresponds to  $\approx \ell/250$  (4 500/18,2).

When the long term deflections are to be estimated it is more relevant to refer to the quasi-permanent load case:

$$q_d = g_k + \sum \psi_{2,i} q_{k,i}$$

For residential building loads the value of  $\psi_2$  is 0,3, see Volume 2: Table 2.2.

The final deflection from the self-weight can either be determined by using the value:

$$E_{\text{mean,fin}} = \frac{E_{\text{mean}}}{1 + k_{\text{def}}}$$

in:

$$w = \frac{5q\ell^4}{384 EI}$$

$k_{\text{def}} = 0,6$  is given by Volume 2: Table 9.1.

This gives:

$$E_{\text{mean,fin}} = \frac{11000}{1 + 0,6} = 6875 \text{ MPa}$$

$$w_{\text{fin,G}} = \frac{5g\ell^4}{384 E_{\text{mean,fin}} I} = \frac{5 \cdot 0,5 \cdot 10^3 \cdot 0,6 \cdot 4,5^4}{384 \cdot 6875 \cdot 10^6 \cdot 39,93 \cdot 10^{-6}} \cdot 10^3 = 5,8 \text{ mm}$$

Alternatively this deflection can be given by:

$$w_{\text{fin,G}} = w_{\text{inst,G}} (1 + k_{\text{def}})$$

$$w_{\text{fin,G}} = 3,6 \cdot (1 + 0,6) = 5,8 \text{ mm}$$

The creep deflection for the variable load is determined from  $\psi_2 q_k$ . Corresponding to the self-weight the deflection for the variable load can be calculated as:

$$w_{\text{fin,Q}} = w_{\text{inst,Q}} (1 + \psi_2 k_{\text{def}})$$

$$w_{\text{fin,Q}} = 14,6 \cdot (1 + 0,3 \cdot 0,6) = 17 \text{ mm}$$

In total the final long term deflection (including the short term effects) corresponds to  $\approx \ell/200$  (4 500/22,8).

The acceptance for a certain limit has to be determined based upon the use of the structural element. In some cases the value of  $\ell/200$  can be a problem but in some cases it may be acceptable. It must be noticed that the short term effects of the characteristic load is included in  $\ell/200$ .

### Example 7.2 Serviceability check of floor with timber joists – vibration

A timber floor in a residential building has a clear span of 3,7 m between supports. The structure comprises 45 × 220 mm timber joists spaced at 0,4 m centres, strength class C18 in service class 1.

The flooring consists of 18 mm thick OSB/3 panel and it is nailed to the joists. In this example the OSB panel is oriented with the main direction of the strands parallel to the timber joists.

The floor width is 4,4 m and the floor mass, based on permanent loading only, is 35 kg/m<sup>2</sup>.

**Carry out a design check to confirm that the vibration behaviour of the floor will be acceptable**, see Volume 2: Section 9.4 or Eurocode 5: Section 7.3.

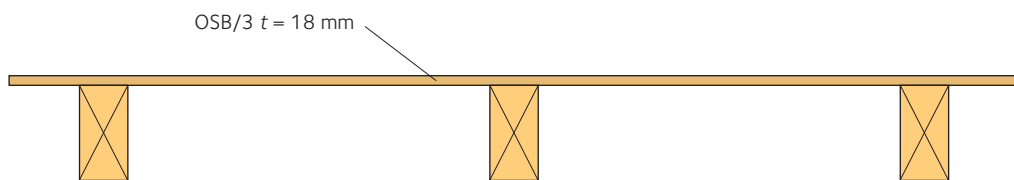


Figure 7.1

Moment of inertia  $I$ :

$$I_{\text{joist}} = \frac{bh^3}{12} = \frac{0,045 \cdot 0,22^3}{12} = 39,93 \cdot 10^{-6} \text{ m}^4$$

Second moment of area of flooring/meter width  $I_f$ :

$$I_f = \frac{b_f t^3}{12} = \frac{1 \cdot 0,018^3}{12} = 4,86 \cdot 10^{-7} \text{ m}^4$$

Width of floor structure,  $B = 4,4 \text{ m}$

#### Strength class C18

Mean modulus of elasticity of the joists parallel to the grain,  $E_{0,\text{mean}} = 9\,000 \text{ MPa}$

#### OSB/3

Mean modulus of elasticity of flooring parallel to the grain,  $E_m = 4\,930 \text{ MPa}$

Mass of the floor (kg/m<sup>2</sup>),  $m = 35 \text{ kg/m}^2$

**a. Check the fundamental frequency of the floor** (see Volume 1: Equation 7.25):

$$f_1 = \frac{\pi}{2\ell^2} \sqrt{\frac{(EI)_\ell}{m}}$$

$$f_1 = \frac{\pi}{2 \cdot 3,7^2} \sqrt{\frac{\left(\frac{9000 \cdot 10^6 \cdot 39,93 \cdot 10^{-6}}{0,4}\right)}{35}} = 18,4$$

The fundamental frequency of the floor is acceptable as it is greater than 8 Hz and the equations given in Eurocode 5 can be used for a vibration check.

**b. Check the static deflection of the floor according to the point load criterion in Volume 1: Equation 7.19:**

Maximum allowable deflection under a 1 kN point load is  $a = 1,5$  mm

$$w = \frac{P\ell^3}{48EI}$$

Check one single joist:

$$P = 1\,000 \text{ N}$$

$$w = \frac{1000 \cdot 3,7^3}{48 \cdot 9000 \cdot 10^6 \cdot 39,93 \cdot 10^{-6}} \cdot 10^3 = 2,9 \text{ mm}$$

Since this is not good enough a check is made with regard to the effect of the rigidity in both floor directions (see Volume 1: Equation 7.20 – 7.22):

$$w = \kappa \frac{P\ell^3}{48EI}$$

Where the load distribution factor  $\kappa$  is determined by:

$$\kappa = \begin{cases} -4,7\beta^2 + 2,9\beta + 0,4 & \text{when } 0 \leq \beta < 0,3 \\ 0,8 + 0,2\beta & \text{when } 0,3 \leq \beta \leq 1,0 \end{cases}$$

with:

$$\beta = \frac{(EI)_\ell}{(EI)_B} \left(\frac{s}{\ell}\right)^4$$

$$\beta = \frac{\left(\frac{9000 \cdot 39,93}{0,4}\right)}{(4930 \cdot 0,486)} \left(\frac{400}{3700}\right)^4 = 0,051$$

$$\kappa = -4,7 \cdot 0,051^2 + 2,9 \cdot 0,051 + 0,4 = 0,53$$

We then get, after load distribution by means of the flooring, a deflection under the point load:

$$w = 0,53 \cdot 2,9 \text{ mm} = 1,6 \text{ mm}$$

This means that the static check not is totally fulfilled but all the same we go further with the control. We could also choose to change the timber quality.

The following should also be valid:

$$v \leq b^{(f_1 \zeta^{-1})}$$

where  $v$  is the unit impulse velocity response ( $\text{m}/(\text{Ns}^2)$ ).  $\zeta$  is the modal damping ratio which normally can be set to 0,01. The constant  $b$  for the control of unit impulse response can be set to 100 (see Volume 2: Section 9.4):

$$v \leq b^{(f_1 \zeta^{-1})} = 100^{(18,4 \cdot 0,01^{-1})} = 0,023$$

The unit impulse velocity response is given by:

$$v = \frac{4(0,4 + 0,6n_{40})}{mB\ell + 200}$$

where  $n_{40}$  is the number of first order modes with natural frequencies up to 40 Hz and  $b$  is the floor width. The value of  $n_{40}$  may be calculated from:

$$n_{40} = \left\{ \left( \left( \frac{40}{f_1} \right)^2 - 1 \right) \left( \frac{B}{\ell} \right)^4 \frac{(EI)_\ell}{(EI)_B} \right\}^{0,25}$$

$$n_{40} = \left\{ \left( \left( \frac{40}{18,4} \right)^2 - 1 \right) \left( \frac{4400}{3700} \right)^4 \frac{\left( \frac{9900 \cdot 39,93}{0,4} \right)}{(4930 \cdot 0,486)} \right\}^{0,25} = 7,5$$

The unit impulse velocity response is then:

$$v = \frac{4(0,4 + 0,6 \cdot 7,5)}{35 \cdot 4,4 \cdot 3,7 + 200} = 0,025$$

- **This value is higher than the limit value 0,023  $\text{m}/(\text{Ns}^2)$ . The chosen floor design should therefore be adjusted in order to avoid problems with vibration.**

### Example 7.3 Serviceability check of floor with glulam elements – vibration

A timber floor in a residential multi-storey building is made up of a OSB/3 panel as upper flange and with glulam beams class GL28cs as webs and bottom flanges. The OSB panel has a thickness of 25 mm and is oriented with the main direction of the strands parallel to the glulam beams in this example. The web has a dimension of  $42 \times 315$  mm and a lower flange with dimension  $225 \times 42$  mm. The beams are spaced at 0,6 m centres.

The floor system can be assumed to have full interaction. The floor has a length of 5,5 m (in the direction of the webs) and a width of 8,0 m. The floor is simply supported.

The mass of the floor is  $45 \text{ kg/m}^2$ .

**Carry out a design check to confirm that the vibration behaviour of the floor will be acceptable, see Volume 2: Section 9.4 or Eurocode 5: Section 7.3.**

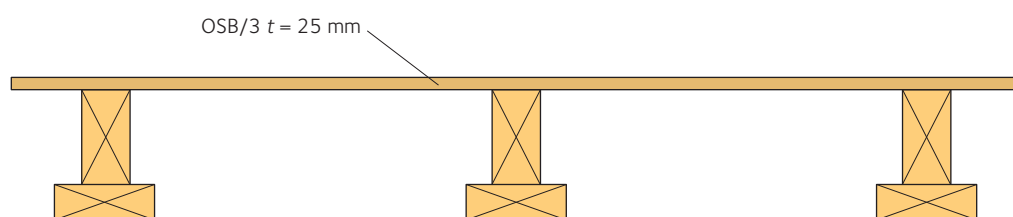


Figure 7.2

First transform the cross section to a fictive section consisting of glulam, GL28cs.

The effective width  $b_{\text{eff}}$  of the OSB can be determined to  $0,15 \ell$  according to Volume 1: Table 5.1 or Eurocode 5: Table 9.1. Since  $\ell = 5,5$  m,  $b_{\text{eff}} = 0,825$  m which is larger than 0,6 m. This means that the whole width of the OSB flange can be used in the calculation of the moment of inertia  $I$ .

Elastic modulus for GL28cs	$E_{0,\text{mean}} = 12\,500 \text{ MPa}$
Elastic modulus for OSB	$E_m = 4\,930 \text{ MPa}$

The fictitious width of the flange is then given by:

$$b_{f_u,\text{fic}} = \frac{E_m}{E_{0,\text{mean}}} b_u = \frac{4930}{12500} \cdot 600 = 237 \text{ mm}$$

Area of the cross-section:

$$A_c = b_{f_l} \cdot h_\ell + b_w \cdot h_w + b_{f_u,\text{fic}} \cdot h_u = 225 \cdot 42 + 42 \cdot 315 + 237 \cdot 25 = 28596 \text{ mm}^2$$

Centre of gravity (from the lower edge):

$$x_{tp} = \frac{b_{f_l} \cdot h_l \cdot \frac{h_l}{2} + b_w \cdot h_w \cdot \left( h_l + \frac{h_w}{2} \right) + b_{f_u, fic} \cdot h_u \cdot \left( h_l + h_w + \frac{h_u}{2} \right)}{A_c} =$$

$$\frac{225 \cdot 42 \cdot \frac{42}{2} + 42 \cdot 315 \cdot \left( 42 + \frac{315}{2} \right) + 237 \cdot 25 \cdot \left( 42 + 315 + \frac{25}{2} \right)}{28596} = 176 \text{ mm}$$

Second moment of area of the lower flange:

$$I_{f_l} = \frac{b_{f_l} h_l^3}{12} + b_{f_l} h_l \cdot \left[ x_{tp} - \left( \frac{h_{f_l}}{2} \right) \right]^2 =$$

$$\frac{0,225 \cdot 0,042^3}{12} + 0,225 \cdot 0,042 \cdot \left[ 0,176 - \left( \frac{0,042}{2} \right) \right]^2 = 227 \cdot 10^{-6} \text{ m}^4$$

Second moment of area of the upper flange:

$$I_{f_u} = \frac{b_{f_u} h_u^3}{12} + b_{f_u} h_u \cdot \left[ x_{tp} - \left( h_{f_l} + h_w + \frac{h_{f_u}}{2} \right) \right]^2 =$$

$$\frac{0,237 \cdot 0,025^3}{12} + 0,237 \cdot 0,025 \cdot \left[ 0,176 - \left( 0,042 + 0,315 + \frac{0,025}{2} \right) \right]^2 = 222 \cdot 10^{-6} \text{ m}^4$$

Second moment of area of the web:

$$I_w = \frac{b_w h_w^3}{12} + b_w h_w \cdot \left[ x_{tp} - \left( h_{f_l} + \frac{h_w}{2} \right) \right]^2 =$$

$$\frac{0,042 \cdot 0,315^3}{12} + 0,042 \cdot 0,315 \cdot \left[ -0,176 + 0,042 + \frac{0,315}{2} \right]^2 = 117 \cdot 10^{-6} \text{ m}^4$$

Second moment of area:

$$I = I_{f_l} + I_{f_u} + I_w = 227 \cdot 10^{-6} + 222 \cdot 10^{-6} + 117 \cdot 10^{-6} = 566 \cdot 10^{-6} \text{ m}^4$$

Second moment of area of flooring/meter width:

$$I_f = \frac{b_l h_{f_u}^3}{12} = \frac{1 \cdot 0,025^3}{12} = 1,3 \cdot 10^{-6} \text{ m}^4$$

Width of floor structure,  $B = 8 \text{ m}$

Mass of the floor ( $\text{kg/m}^2$ ),  $m = 45 \text{ kg/m}^2$

**a. Check fundamental frequency of the floor:**

$$f_1 = \frac{\pi}{2\ell^2} \sqrt{\frac{(EI)_\ell}{m}}$$

$$f_1 = \frac{\pi}{2 \cdot 5,5^2} \sqrt{\frac{\left(\frac{12500 \cdot 10^6 \cdot 566 \cdot 10^{-6}}{0,6}\right)}{45}} = 26,6$$

The fundamental frequency of the floor is acceptable as it is greater than 8 Hz and the equations given in Volume 1: Section 7.3.3.1 can be used for a vibration check.

**b. Check the static deflection of the floor:**

Maximum allowable deflection under a 1 kN point load is  $a = 1,5$  mm:

$$w = \frac{P\ell^3}{48EI}$$

Check one single beam:

$$P = 1\,000 \text{ N}$$

$$w = \frac{1000 \cdot 5,5^3}{48 \cdot 12,5 \cdot 10^9 \cdot 566 \cdot 10^{-6}} \cdot 1000 = 0,5 \text{ mm}$$

The deflection of the floor is less than the allowable value.

The following should also be valid:

$$v \leq b^{(f_i \zeta - 1)}$$

where  $v$  is the unit impulse velocity response ( $\text{m}/(\text{Ns}^2)$ ).  $\zeta$  is the modal damping ratio which normally can be set to 0,01. The constant  $b$  for the control of unit impulse response can be set to 100 (see Volume 2: Section 9.4):

$$v \leq b^{(f_i \zeta - 1)} = 100^{(26,6 \cdot 0,01 - 1)} = 0,034$$

The unit impulse velocity response is given by:

$$v = \frac{4(0,4 + 0,6n_{40})}{mB\ell + 200}$$



where  $n_{40}$  is the number of first order modes with natural frequencies up to 40 Hz and  $B$  is the floor width. The value of  $n_{40}$  may be calculated from:

$$n_{40} = \left\{ \left( \left( \frac{40}{f_1} \right)^2 - 1 \right) \left( \frac{B}{\ell} \right)^4 \frac{(EI)_\ell}{(EI)_B} \right\}^{0,25}$$

$$n_{40} = \left\{ \left( \left( \frac{40}{26,6} \right)^2 - 1 \right) \left( \frac{8000}{5500} \right)^4 \frac{\left( \frac{12500 \cdot 566}{0,6} \right)}{(4930 \cdot 1,3)} \right\}^{0,25} = 10,1$$

The unit impulse velocity response is then:

$$v = \frac{4(0,4 + 0,6 \cdot 10,1)}{45 \cdot 8 \cdot 5,5 + 200} = 0,012$$

- **As the unit impulse velocity response of the floor is less than the allowable value it is acceptable. The floor has complied with the vibration requirements in Eurocode 5.**

# Symbols

Symbols used in SS-EN 1995-1-1.

Symbol	Explanation
<b>Latin upper case letters</b>	
$A$	Cross-sectional area
$A_{ef}$	Effective area of the total contact surface between a punched metal plate fastener and the timber; effective total contact surface perpendicular to the grain
$A_f$	Cross-sectional area of flange
$A_{net,t}$	Net cross-sectional area perpendicular to the grain
$A_{net,v}$	Net shear area parallel to the grain
$C$	Spring stiffness
$E_{0,05}$	Fifth percentile value of modulus of elasticity
$E_d$	Design value of modulus of elasticity
$E_{mean}$	Mean value of modulus of elasticity
$E_{mean,fin}$	Final mean value of modulus of elasticity
$F$	Force
$F_{A,Ed}$	Design force acting on a punched metal plate fastener at the centroid of the effective area
$F_{A,min,d}$	Minimum design force acting on a punched metal plate fastener at the centroid of the effective area
$F_{ax,Ed}$	Design axial force on fastener
$F_{ax,Rd}$	Design value of axial withdrawal capacity of the fastener
$F_{ax,Rk}$	Characteristic axial withdrawal capacity of the fastener
$F_c$	Compressive force
$F_d$	Design force
$F_{d,ser}$	Design force at the serviceability limit state
$F_{f,Rd}$	Design load-carrying capacity per fastener in wall diaphragm
$F_{i,c,Ed}$	Design compressive reaction force at end of shear wall
$F_{i,t,Ed}$	Design tensile reaction force at end of shear wall
$F_{i,vert,Ed}$	Vertical load on wall
$F_{i,v,Rd}$	Design racking resistance of panel $i$ or wall $i$
$F_{la}$	Lateral load
$F_{M,Ed}$	Design force from a design moment
$F_t$	Tensile force
$F_{t,Rk}$	Characteristic tensile capacity of a connection
$F_{V,0,Rk}$	Characteristic load-carrying capacity of a connector along the grain
$F_{V,Ed}$	Design shear force per shear plane of fastener; Horizontal design effect on wall diaphragm

$F_{v,Rd}$	Design load-carrying capacity per shear plane per fastener; Design racking load capacity
$F_{v,Rk}$	Characteristic load-carrying capacity per shear plane per fastener
$F_{v,w,Ed}$	Design shear force acting on web
$F_{x,Ed}$	Design value of a force in $x$ -direction
$F_{y,Ed}$	Design value of a force in $y$ -direction
$F_{x,Rd}$	Design value of plate capacity in $x$ -direction
$F_{y,Rd}$	Design value of plate capacity in $y$ -direction
$F_{x,Rk}$	Characteristic plate capacity in $x$ -direction
$F_{y,Rk}$	Characteristic plate capacity in $y$ -direction
$G_{0,05}$	Fifth percentile value of shear modulus
$G_d$	Design value of shear modulus
$G_{mean}$	Mean value of shear modulus
$H$	Overall rise of a truss
$I_f$	Second moment of area of flange
$I_{tor}$	Torsional moment of inertia
$I_z$	Second moment of area about the weak axis
$K_{ser}$	Slip modulus
$K_{ser,fin}$	Final slip modulus
$K_u$	Instantaneous slip modulus for ultimate limit states
$L_{net,t}$	Net width of the cross-section perpendicular to the grain
$L_{net,v}$	Net length of the fracture area in shear
$M_{A,Ed}$	Design moment acting on a punched metal plate fastener
$M_{ap,d}$	Design moment at apex zone
$M_d$	Design moment
$M_{y,Rk}$	Characteristic yield moment of fastener
$N$	Axial force
$R_{90,d}$	Design splitting capacity
$R_{90,k}$	Characteristic splitting capacity
$R_{ax,d}$	Design load-carrying capacity of an axially loaded connection
$R_{ax,k}$	Characteristic load-carrying capacity of an axially loaded connection
$R_{ax,\alpha,k}$	Characteristic load-carrying capacity at an angle $\alpha$ to grain
$R_d$	Design value of a load-carrying capacity
$R_{ef,k}$	Effective characteristic load-carrying capacity of a connection
$R_{iv,d}$	Design racking capacity of a wall
$R_k$	Characteristic load-carrying capacity
$R_{sp,k}$	Characteristic splitting capacity

Source: SS-EN 1995-1-1:2004, 1.6

$R_{to,k}$	Characteristic load-carrying capacity of a toothed plate connector
$R_{v,d}$	Design racking capacity of a wall diaphragm
$V$	Shear force; volume
$V_u, V_\ell$	Shear forces in upper and lower part of beam with a hole
$W_y$	Section modulus about axis $y$
$X_d$	Design value of a strength property
$X_k$	Characteristic value of a strength property
<b>Latin lower case letters</b>	
$a$	Distance
$a_1$	Spacing, parallel to grain, of fasteners within one row
$a_{1,CG}$	Minimum end distance to the centre of gravity of the screw in each timber member
$a_2$	Spacing, perpendicular to grain, between rows of fasteners
$a_{2,CG}$	Minimum edge distance to the centre of gravity of the screw in each timber member
$a_{3,c}$	Distance between fastener and unloaded end
$a_{3,t}$	Distance between fastener and loaded end
$a_{4,c}$	Distance between fastener and unloaded edge
$a_{4,t}$	Distance between fastener and loaded edge
$a_{bow}$	Maximum bow of truss member
$a_{bow,perm}$	Maximum permitted bow of truss member
$a_{dev}$	Maximum deviation of truss
$a_{dev,perm}$	Maximum permitted deviation of truss
$b$	Width
$b_i$	Width of panel $i$ or wall $i$
$b_{net}$	Clear distance between studs
$b_w$	Web width
$d$	Diameter; outer thread diameter
$d_i$	Diameter of centre hole of connector; inner thread diameter
$d_c$	Connector diameter
$d_{ef}$	Effective diameter
$d_h$	Head diameter of connector
$f_{h,i,k}$	Characteristic embedment strength of timber member $i$
$f_{a,0,0}$	Characteristic anchorage capacity per unit area for $\alpha = 0^\circ$ och $\beta = 0^\circ$
$f_{a,90,90}$	Characteristic anchorage capacity per unit area for $\alpha = 90^\circ$ och $\beta = 90^\circ$
$f_{a,\alpha,\beta,k}$	Characteristic anchorage strength
$f_{ax,k}$	Characteristic withdrawal parameter for nails
$f_{c,0,d}$	Design compressive strength along the grain
$f_{c,w,d}$	Design compressive strength of web
$f_{f,c,d}$	Design compressive strength of flange

$f_{c,90,k}$	Characteristic compressive strength perpendicular to grain
$f_{f,t,d}$	Design tensile strength of flange
$f_{h,k}$	Characteristic embedment strength
$f_{head,k}$	Characteristic pull-through parameter for nails
$f_1$	Fundamental frequency
$f_{m,k}$	Characteristic bending strength
$f_{m,y,d}$	Design bending strength about the principal $y$ -axis
$f_{m,z,d}$	Design bending strength about the principal $z$ -axis
$f_{m,\alpha,d}$	Design bending strength at an angle $\alpha$ to the grain
$f_{t,0,d}$	Design tensile strength along the grain
$f_{t,0,k}$	Characteristic tensile strength along the grain
$f_{t,90,d}$	Design tensile strength perpendicular to the grain
$f_{t,w,d}$	Design tensile strength of the web
$f_{u,k}$	Characteristic tensile strength of bolts
$f_{v,0,d}$	Design panel shear strength
$f_{v,\alpha,\alpha,k}$	Characteristic withdrawal strength at an angle $\alpha$ to grain
$f_{v,\alpha,\alpha,k}$	Characteristic withdrawal strength perpendicular to grain
$f_{v,d}$	Design shear strength
$h$	Depth; height of wall
$h_{ap}$	Depth of the apex zone
$h_d$	Hole depth
$h_e$	Embedment depth; loaded edge distance
$h_{ef}$	Effective depth
$h_{f,c}$	Depth of compression flange
$h_{f,t}$	Depth of tension flange
$h_{rl}$	Distance from lower edge of hole to bottom of member
$h_{ru}$	Distance from upper edge of hole to top of member
$h_w$	Web depth
$i$	Notch inclination
$k_{c,y}, k_{c,z}$	Instability factor
$k_{cr}$	Crack factor for shear resistance
$k_{crit}$	Factor used for lateral buckling
$k_d$	Dimension factor for panel
$k_{def}$	Deformation factor
$k_{dis}$	Factor taking into account the distribution of stresses in an apex zone
$k_{f,1}, k_{f,2}, k_{f,3}$	Modification factors for bracing resistance
$k_h$	Depth factor
$k_{i,q}$	Uniformly distributed load factor

Source: SS-EN 1995-1-1:2004, 1.6

$k_m$	Factor considering re-distribution of bending stresses in a cross-section
$k_{mod}$	Modification factor for duration of load and moisture content
$k_n$	Sheathing material factor
$k_r$	Reduction factor
$k_{R,red}$	Reduction factor for load-carrying capacity
$k_s$	Fastener spacing factor; modification factor for spring stiffness
$k_{s,red}$	Reduction factor for spacing
$k_{shape}$	Factor depending on the shape of the cross-section
$k_{sys}$	System strength factor
$k_v$	Reduction factor for notched beams
$k_{vol}$	Volume factor
$k_y$ eller $k_z$	Instability factor
$l_{a,min}$	Minimum anchorage length for a glued-in rod
$l$	Span; contact length
$l_A$	Distance from a hole to the centre of the member support
$l_{ef}$	Effective length; Effective length of distribution
$l_v$	Distance from a hole to the end of the member
$l_z$	Spacing between holes
$m$	Mass per unit area
$n_{40}$	Number of frequencies below 40 Hz
$n_{ef}$	Effective number of fasteners
$p_d$	Distributed load
$q_l$	Equivalent uniformly distributed load
$r$	Radius of curvature
$s$	Spacing
$s_0$	Basic fastener spacing
$r_{in}$	Inner radius
$t$	Thickness
$t_{pen}$	Penetration depth
$u_{creep}$	Creep deformation
$u_{fin}$	Final deformation
$u_{fin,G}$	Final deformation for a permanent action G
$u_{fin,Q,1}$	Final deformation for the leading variable action $Q_1$
$u_{fin,Q,i}$	Final deformation for accompanying variable actions $Q_i$
$u_{inst}$	Instantaneous deformation
$u_{inst,G}$	Instantaneous deformation for a permanent action G
$u_{inst,Q,1}$	Instantaneous deformation for the leading variable action $Q_1$

$u_{inst,Q,i}$	Instantaneous deformation for accompanying variable actions $Q_i$
$w_c$	Precamber
$w_{creep}$	Creep deflection
$w_{fin}$	Final deflection
$w_{inst}$	Instantaneous deflection
$w_{net,fin}$	Net final deflection
$v$	Unit impulse velocity response
<b>Greek lower case letters</b>	
$\alpha$	Angle between the x-direction and the force for a punched metal plate; Angle between the direction of the load and the loaded edge (or end)
$\beta$	Angle between the grain direction and the force for a punched metal plate
$\beta_c$	Straightness factor
$\gamma$	Angle between the x-direction and the timber connection line for a punched metal plate
$\gamma_M$	Partial factor for material properties, also accounting for model uncertainties and dimensional variations
$\lambda_y$	Slenderness ratio corresponding to bending about the y-axis
$\lambda_z$	Slenderness ratio corresponding to bending about the z-axis
$\lambda_{rel,y}$	Relative slenderness ratio corresponding to bending about the y-axis
$\lambda_{rel,z}$	Relative slenderness ratio corresponding to bending about the z-axis
$\rho_k$	Characteristic density
$\rho_m$	Mean density
$\sigma_{c,0,d}$	Design compressive stress along the grain
$\sigma_{c,\alpha,d}$	Design compressive stress at an angle $\alpha$ to the grain
$\sigma_{f,c,d}$	Mean design compressive stress of flange
$\sigma_{f,c,max,d}$	Design compressive stress of extreme fibres of flange
$\sigma_{f,t,d}$	Mean design tensile stress of flange
$\sigma_{f,t,max,d}$	Design tensile stress of extreme fibres of flange
$\sigma_{m,crit}$	Critical bending stress
$\sigma_{m,y,d}$	Design bending stress about the principal y-axis
$\sigma_{m,z,d}$	Design bending stress about the principal z-axis
$\sigma_{m,\alpha,d}$	Design bending stress at an angle $\alpha$ to the grain
$\sigma_N$	Axial stress
$\sigma_{t,0,d}$	Design tensile stress along the grain
$\sigma_{t,90,d}$	Design tensile stress perpendicular to the grain

Source: SS-EN 1995-1-1:2004, 1.6

$\sigma_{w,c,d}$	Design compressive stress of web
$\sigma_{w,t,d}$	Design tensile stress of web
$\tau_d$	Design shear stress
$\tau_{F,d}$	Design anchorage stress from axial force
$\tau_{M,d}$	Design anchorage stress from moment
$\tau_{tor,d}$	Design shear stress from torsion
$\psi_0$	Factor for combination value of a variable action
$\psi_1$	Factor for frequent value of a variable action
$\psi_2$	Factor for quasi-permanent value of a variable action
$\zeta$	Modal damping ratio

Source: SS-EN 1995-1-1:2004, 1.6

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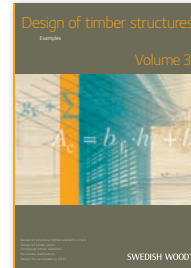
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